# A Mathematician's Notebook: Chronicling the Journey

Tammy L Jones
TammyJones@TLJConsultingGroup.com
Tammy@TexasAndJones.com
Leslie A. Texas
Leslie@TexasAndJones.com

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# WHY THIS JOURNEY?

**The Law of Navigation**

*The 21 Irrefutable Laws of Leadership*

John Maxwell

"Anyone can steer the ship, but it takes a leader to chart the course."

**Begin with the end in mind.**

**Understand what's required to get there.**

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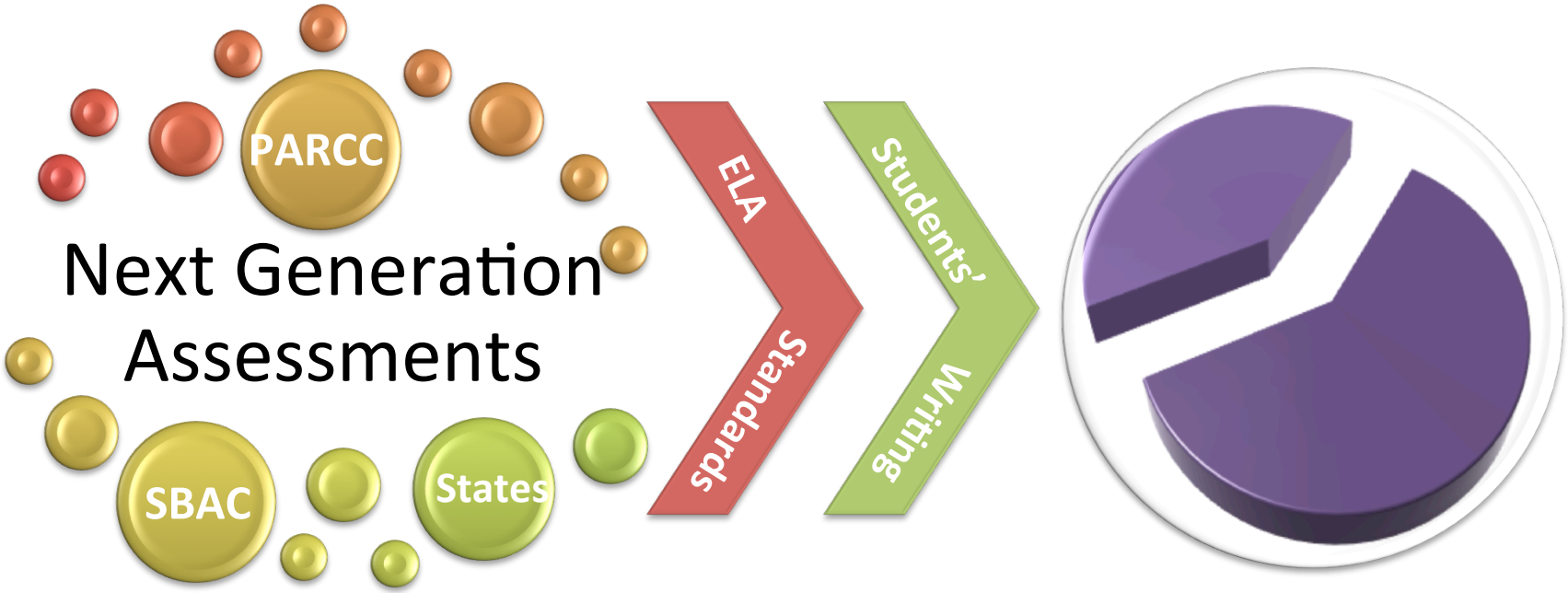


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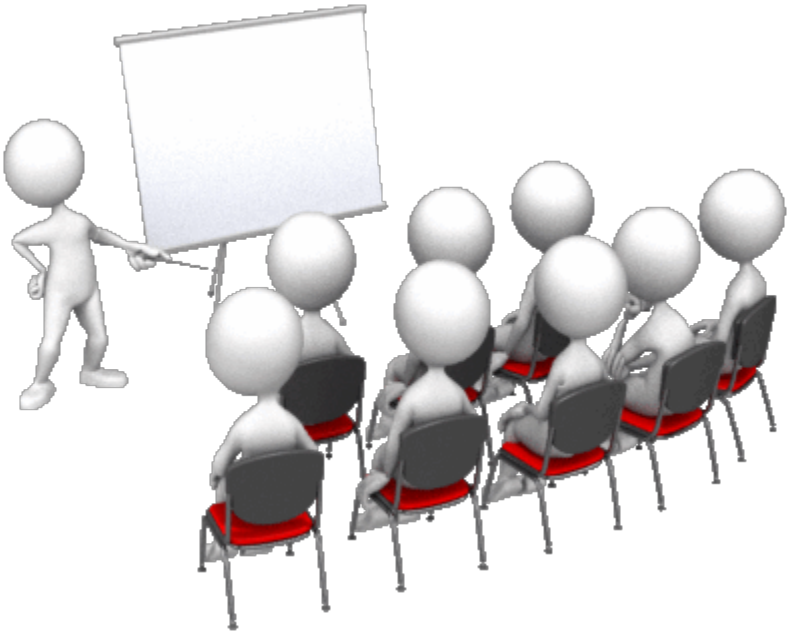

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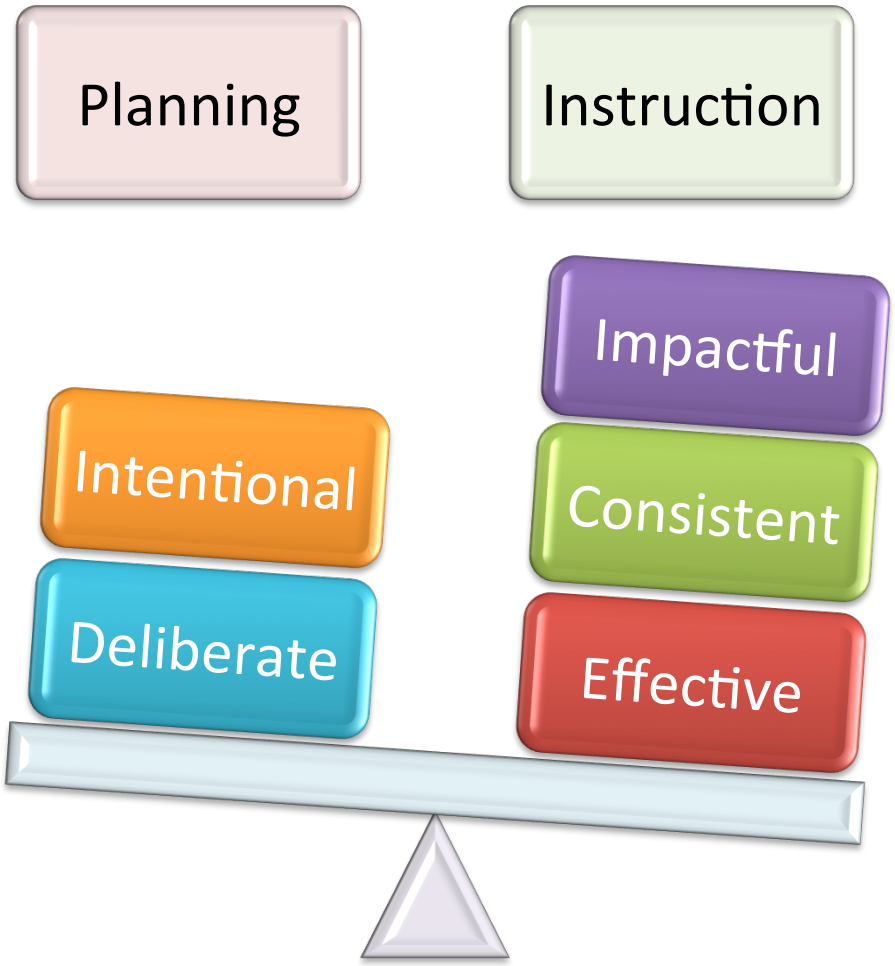


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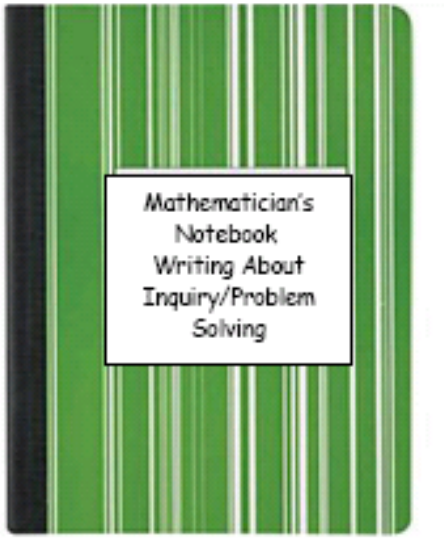
# The Mathematician's Notebook

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"The notebook becomes a place where language, data, and experience operate jointly to form meaning for the student" (Klentschy & De-La Torre).

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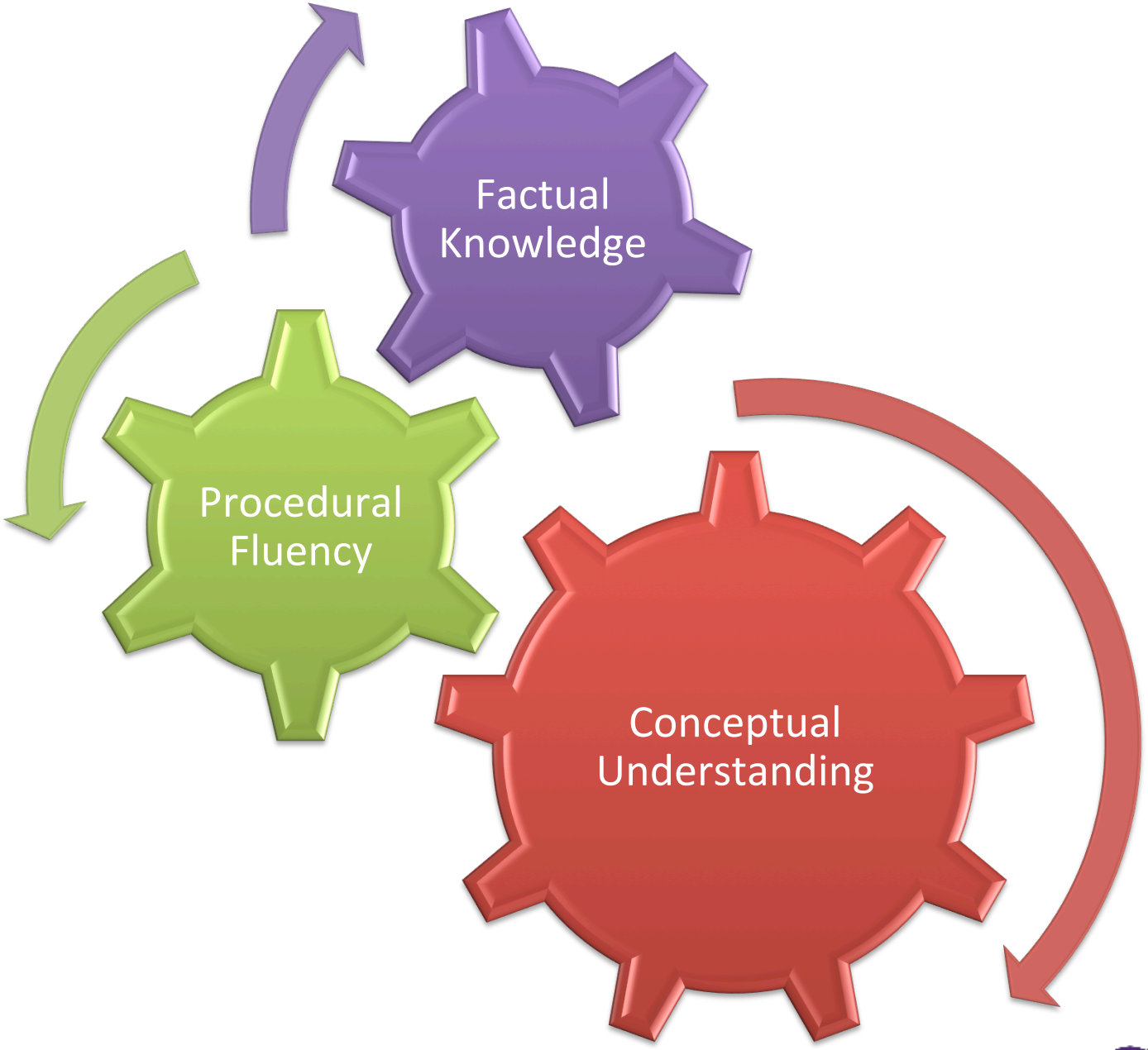
# Learning Mathematics

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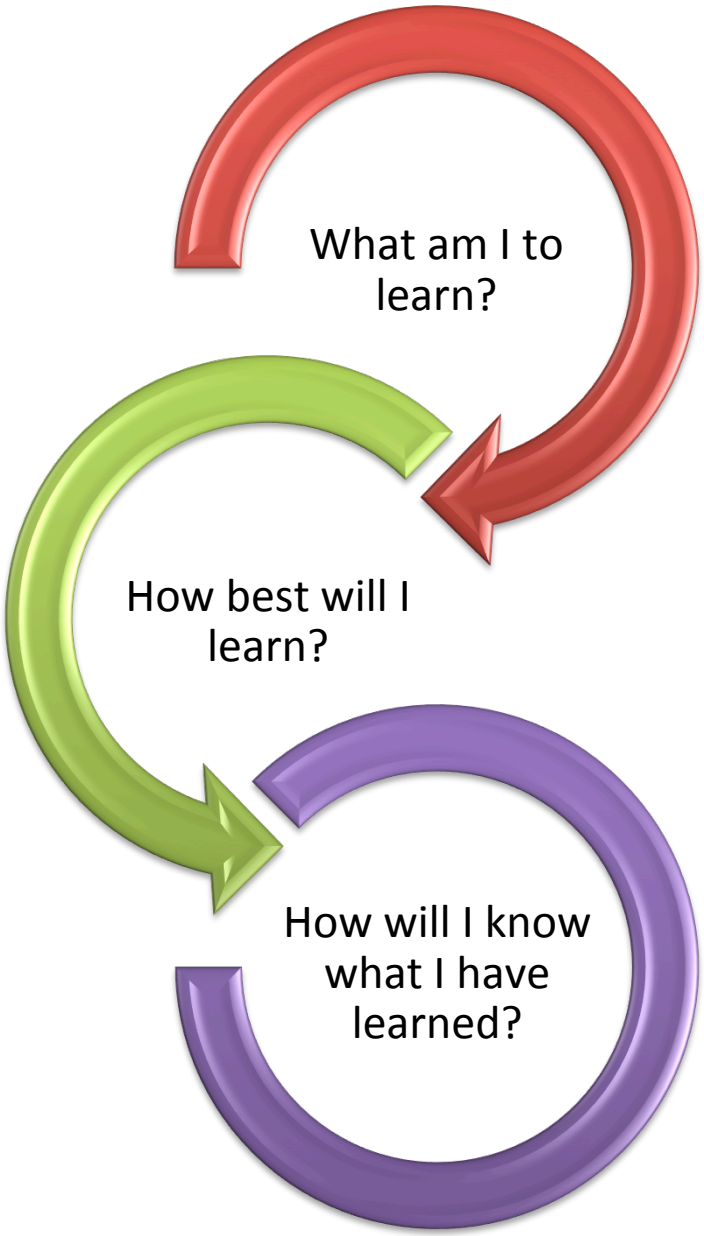
# Learners Constructing Meaning

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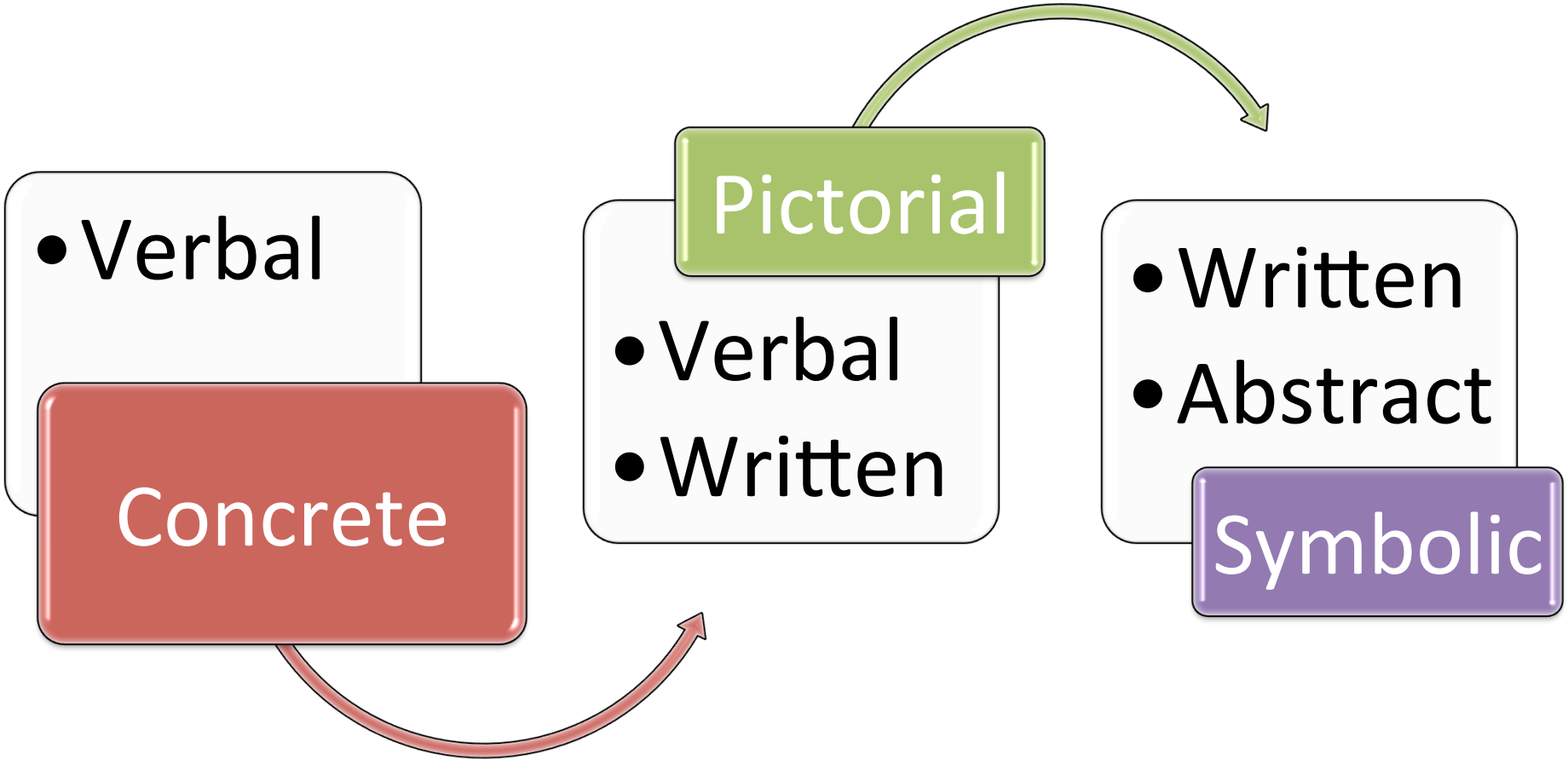
# NCTM…

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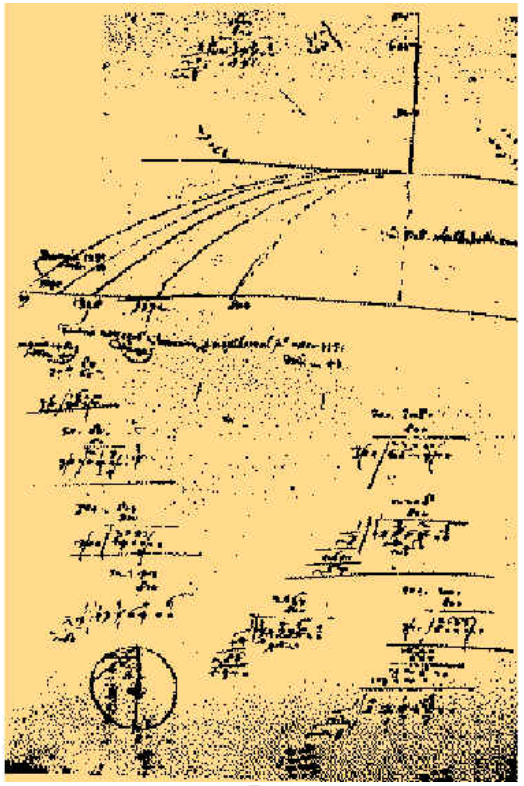
# Why use the Mathematician's Notebook?

"From Galileo to today's scientists and mathematicians …, notebooks have been used to document …discovery. Notebooks are also effective tools in the classroom. They make science and mathematics experiences more meaningful and authentic for students as they observe, record, and reflect on what they've learned."



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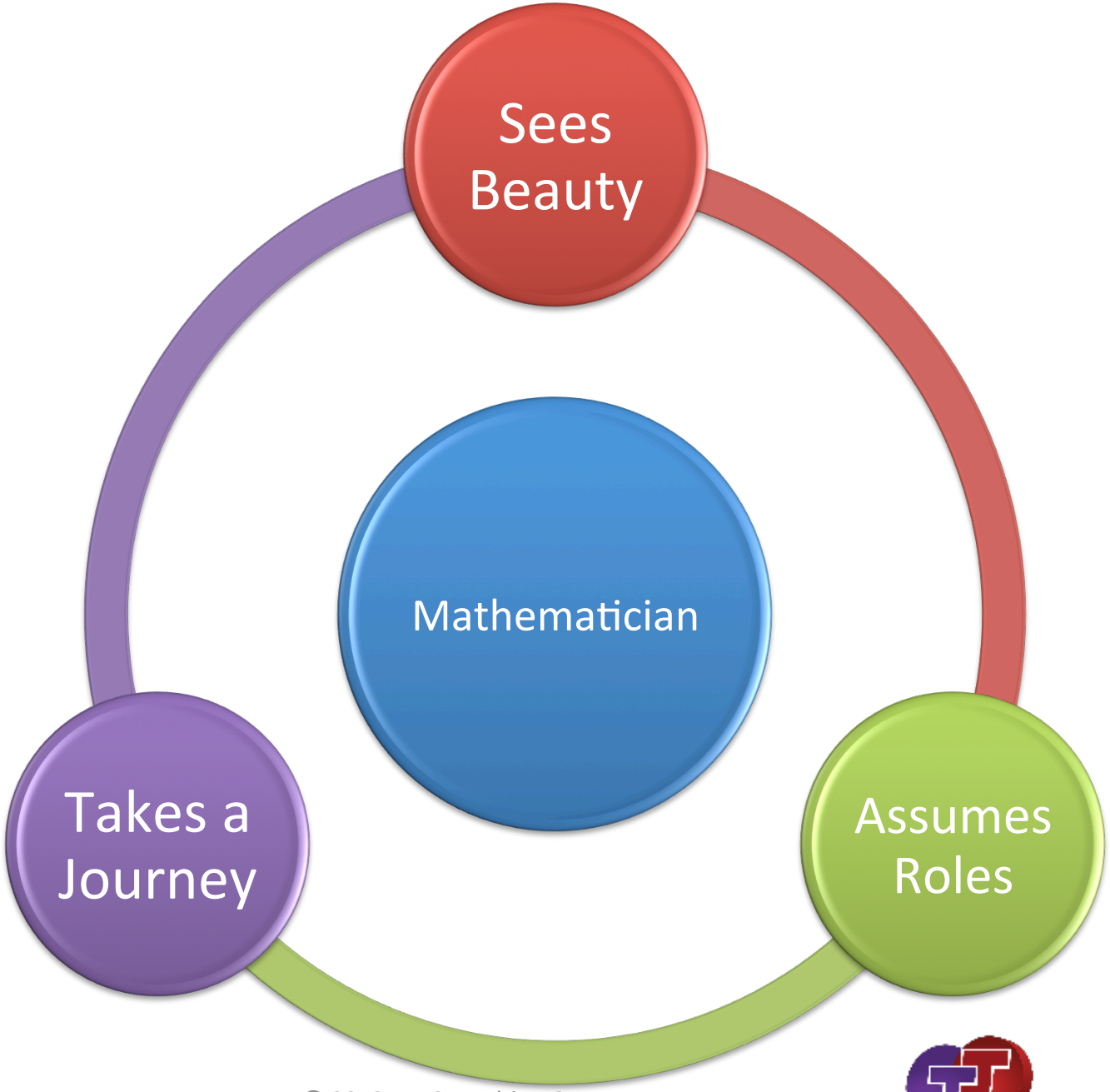
# Mathematician's Notebook

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# Roles for Habits of Mind

Adapted from *Habits of Mind: An Organizing Principle for Mathematics Curriculum*" Al Cuoco, E. Paul Goldenberg, and June Mark, 1996

**Pattern Sniffer**
I look for patterns among a set of numbers or figures or letters.

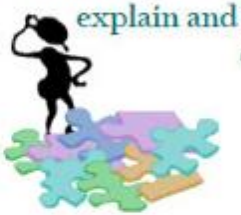
**Experimenter**
I question and then experiment. I record and study results. I perform thought experiments.

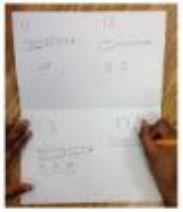
**Describer**
I can describe clearly a problem, a process, or a strategy. I use language appropriate for the audience.

**Tinkerers**
I "play around" and investigate. I can take ideas apart and put them back together. I stick with my work and try a different approach if needed.

**Inventor**
I can invent mathematics. I can model a situation. I use math for fun (in games) as well as for finding a solution to a problem.

**Conjecturer**
I make conjectures and predictions about patterns in numbers, for example. I can explain and support my conjectures.





**Visualizer**
I draw pictures or make diagrams to help me understand a problem. I "see" math. I can understand drawings and diagrams.

**Guesser**
I make clever guesses based on things I already known as well as new patterns and relationships I have observed.

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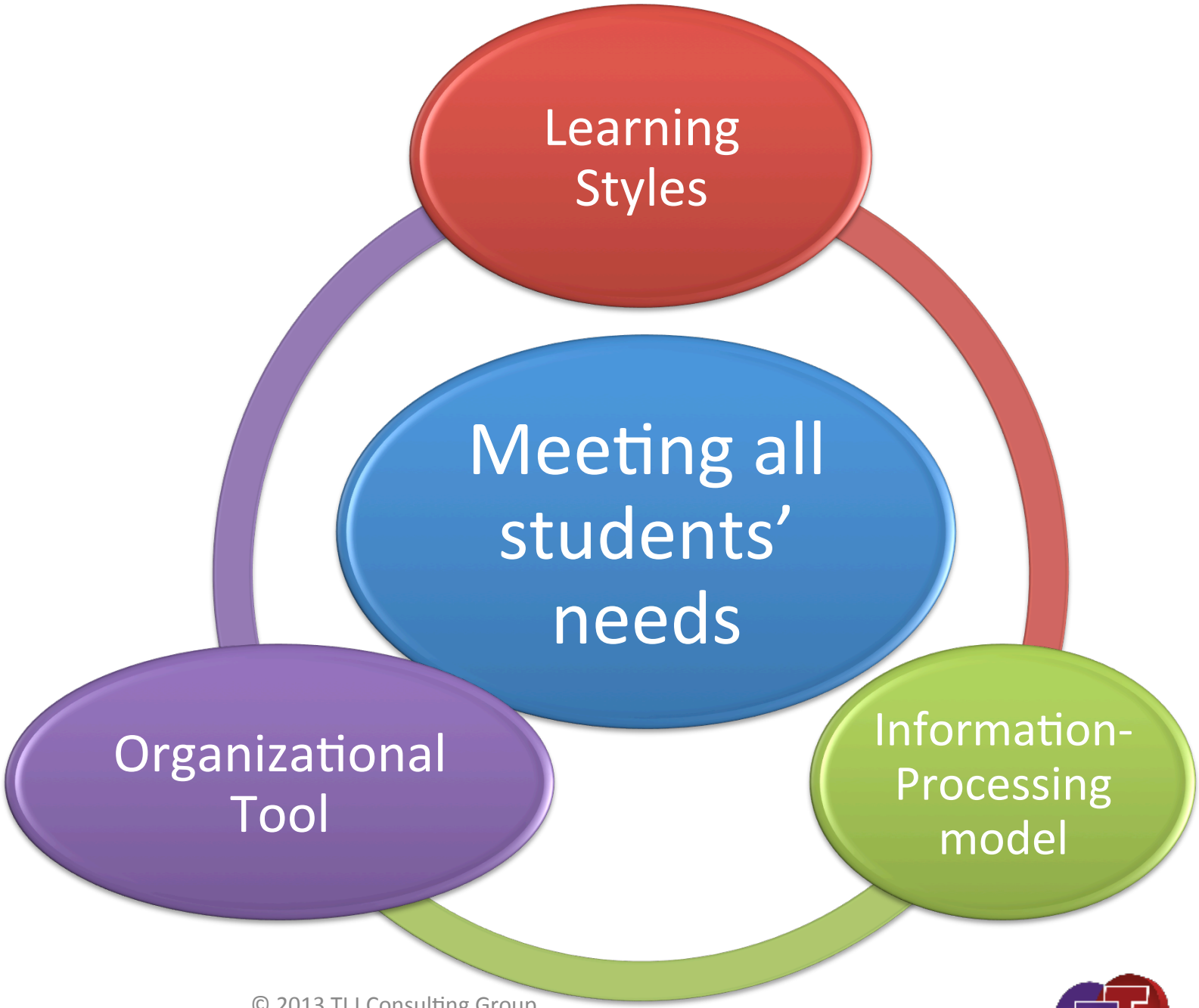
# Research

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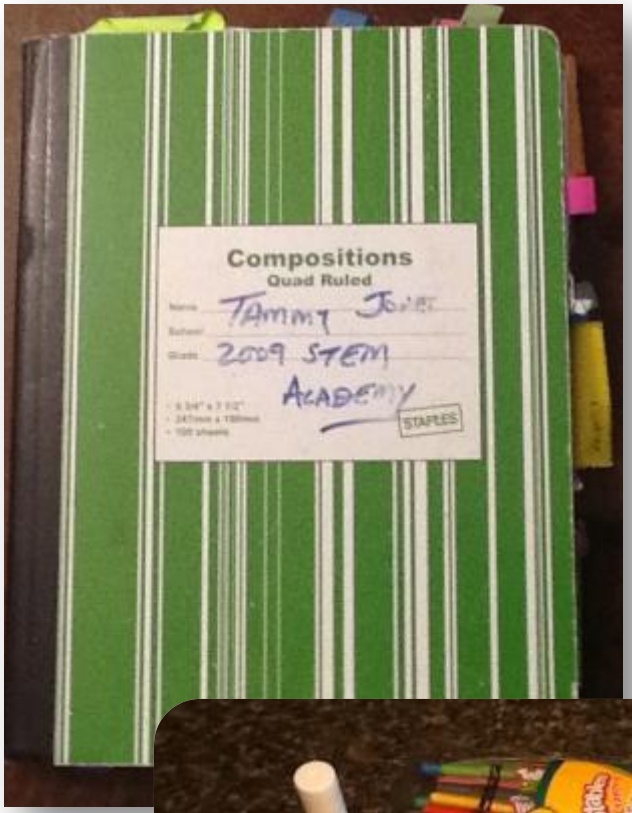
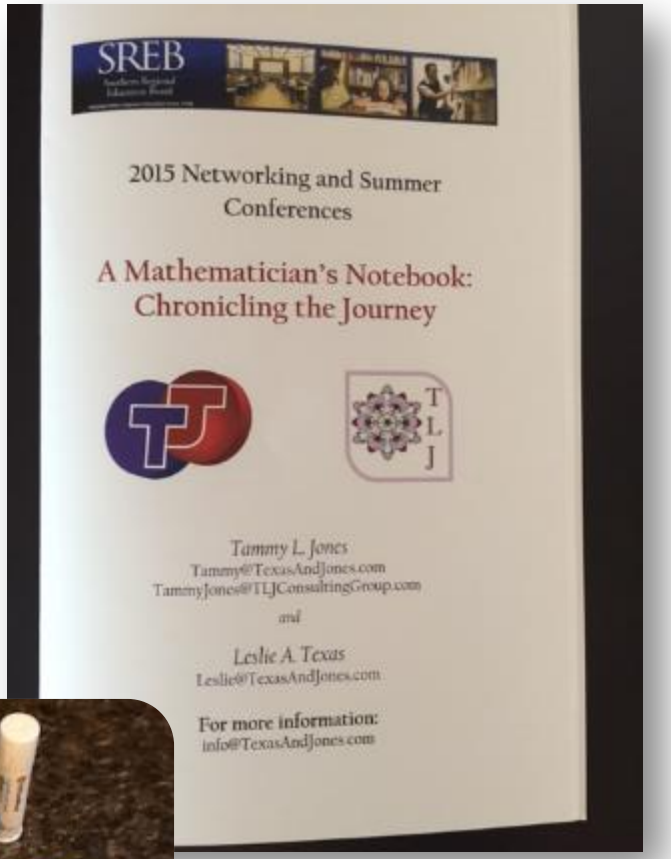
# Materials



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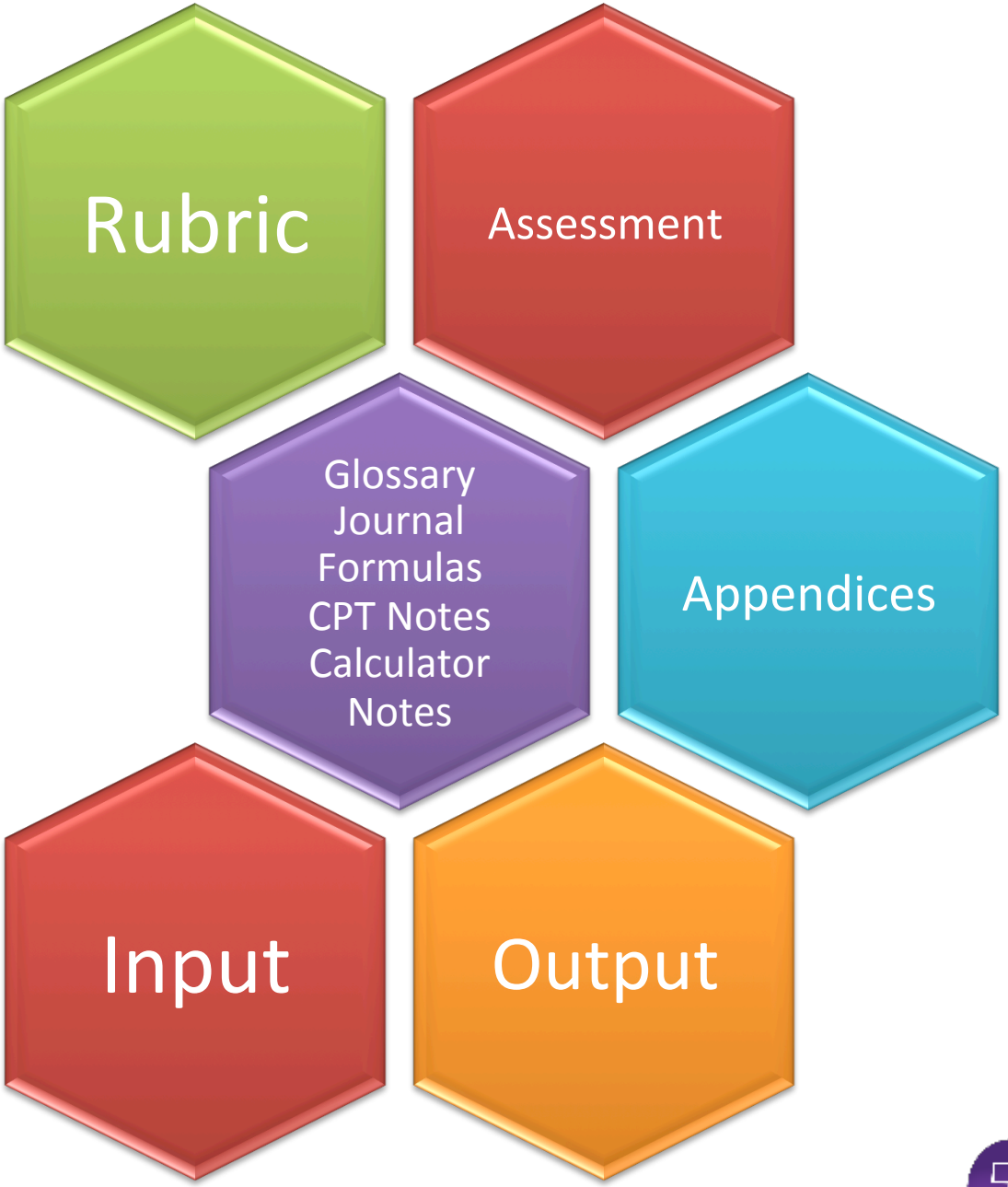
# Components

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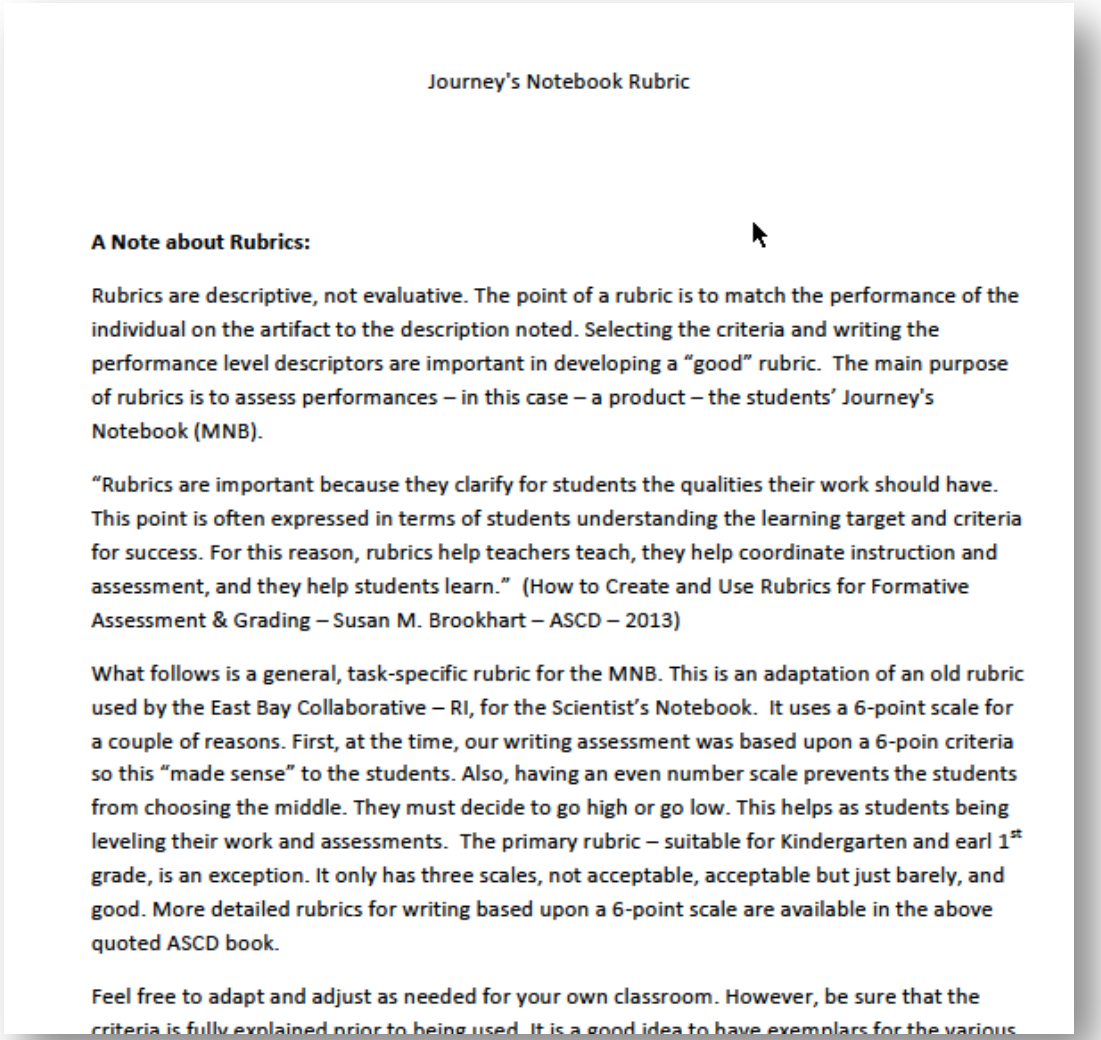
# Rubric



Journey's Notebook Rubric

**A Note about Rubrics:**

Rubrics are descriptive, not evaluative. The point of a rubric is to match the performance of the individual on the artifact to the description noted. Selecting the criteria and writing the performance level descriptors are important in developing a "good" rubric. The main purpose of rubrics is to assess performances – in this case – a product – the students' Journey's Notebook (MNB).

"Rubrics are important because they clarify for students the qualities their work should have. This point is often expressed in terms of students understanding the learning target and criteria for success. For this reason, rubrics help teachers teach, they help coordinate instruction and assessment, and they help students learn." (How to Create and Use Rubrics for Formative Assessment & Grading – Susan M. Brookhart – ASCD – 2013)

What follows is a general, task-specific rubric for the MNB. This is an adaptation of an old rubric used by the East Bay Collaborative – RI, for the Scientist's Notebook. It uses a 6-point scale for a couple of reasons. First, at the time, our writing assessment was based upon a 6-poin criteria so this "made sense" to the students. Also, having an even number scale prevents the students from choosing the middle. They must decide to go high or go low. This helps as students being leveling their work and assessments. The primary rubric – suitable for Kindergarten and earl 1st grade, is an exception. It only has three scales, not acceptable, acceptable but just barely, and good. More detailed rubrics for writing based upon a 6-point scale are available in the above quoted ASCD book.

Feel free to adapt and adjust as needed for your own classroom. However, be sure that the criteria is fully explained prior to being used. It is a good idea to have exemplars for the various

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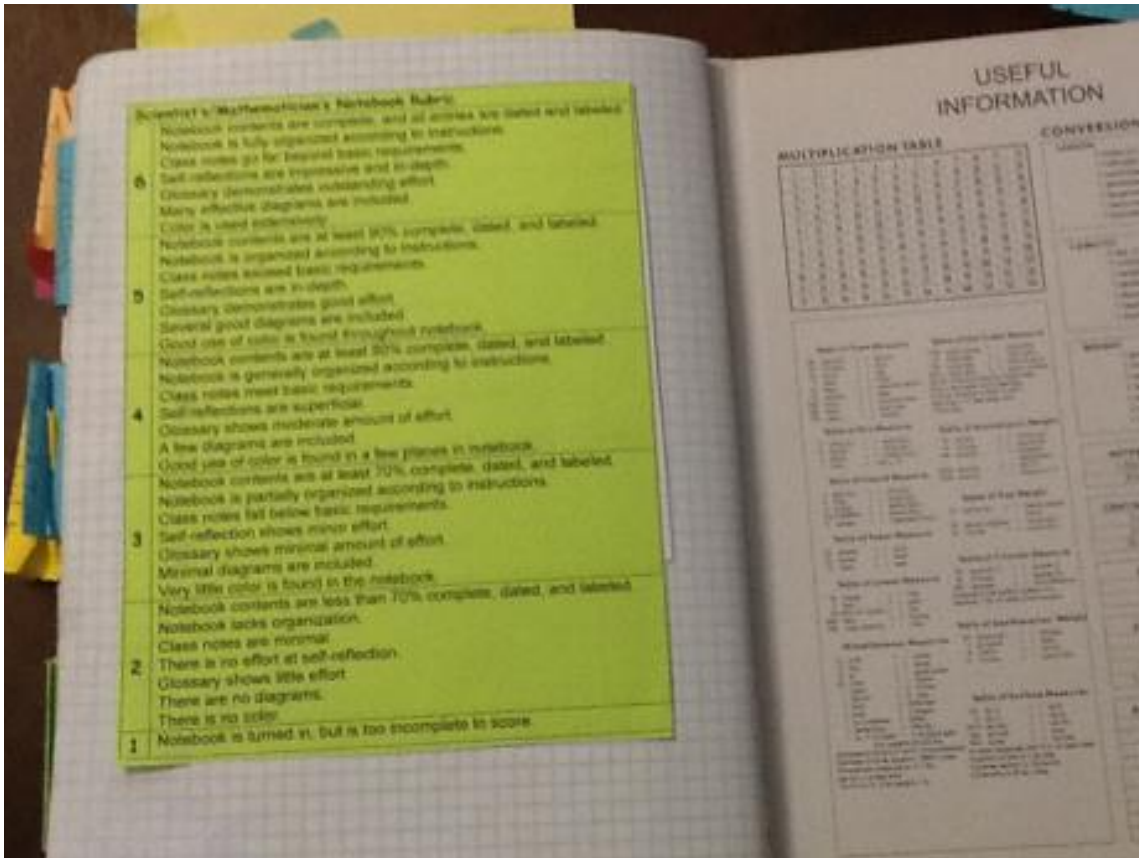
Rubric




NAEP

- Question Tool

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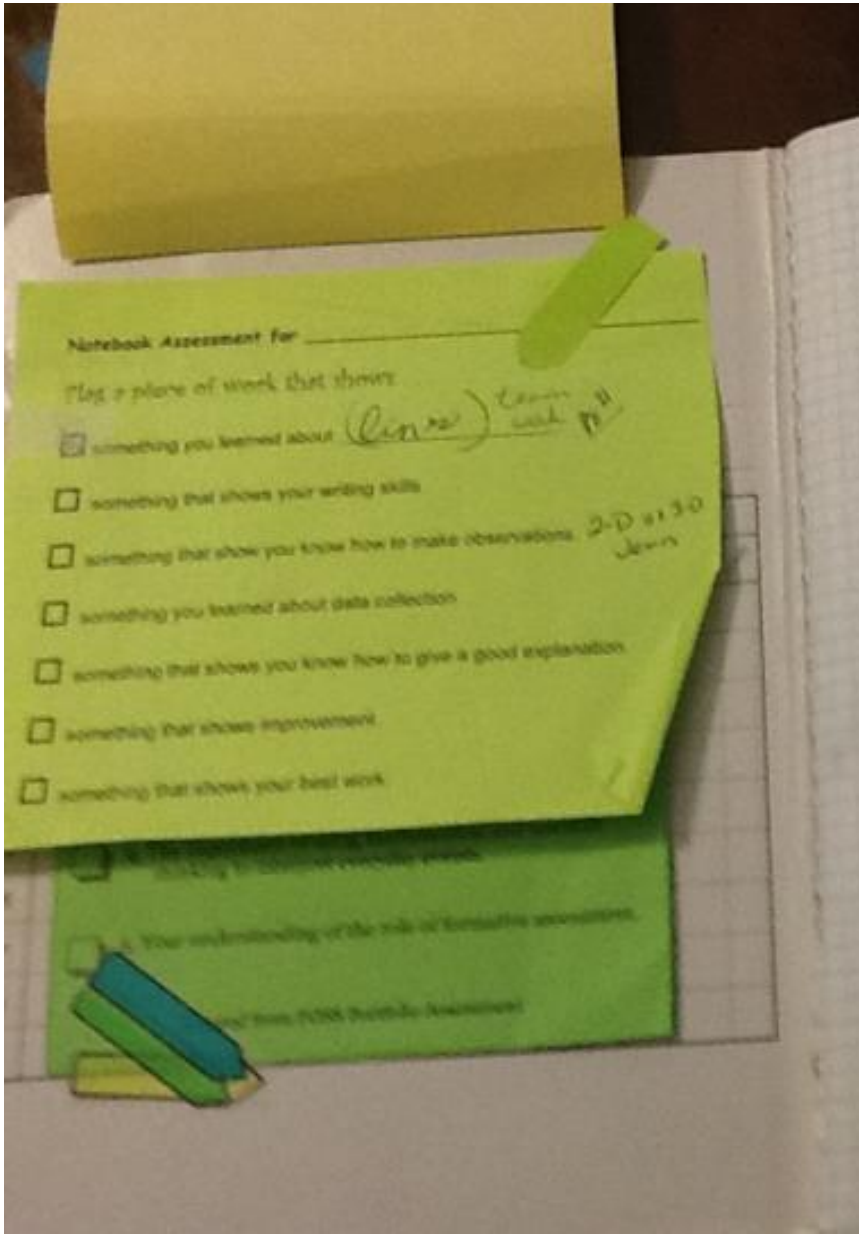
# Assessment

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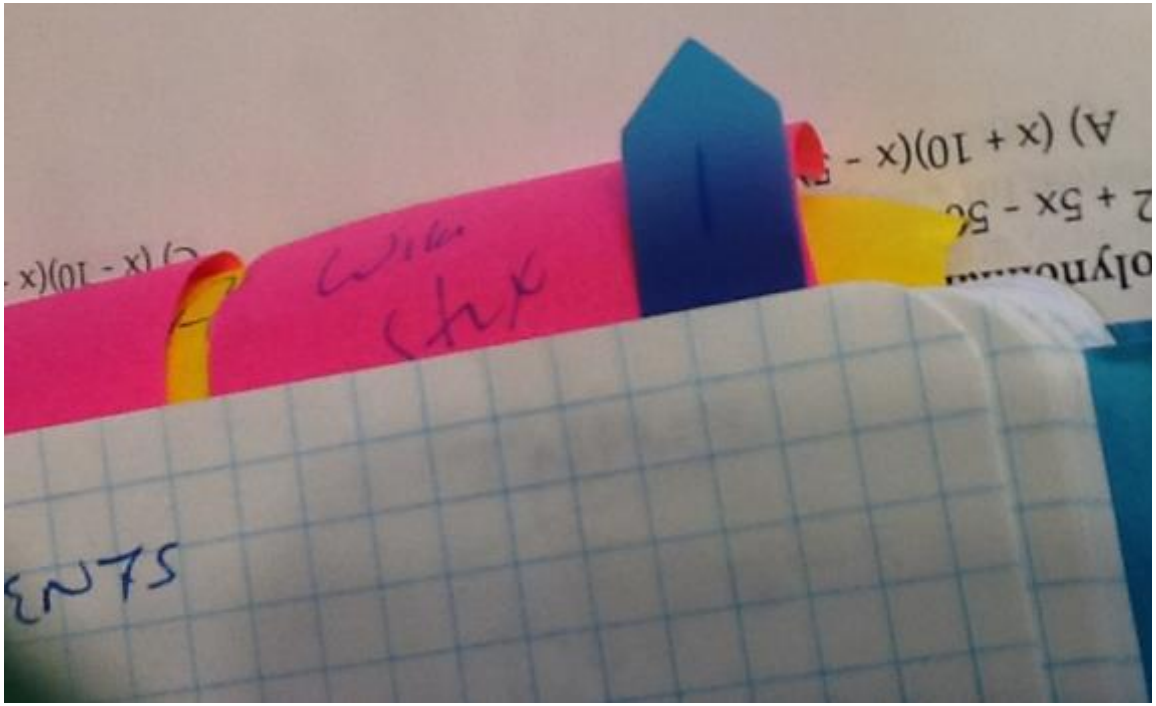
Assessment



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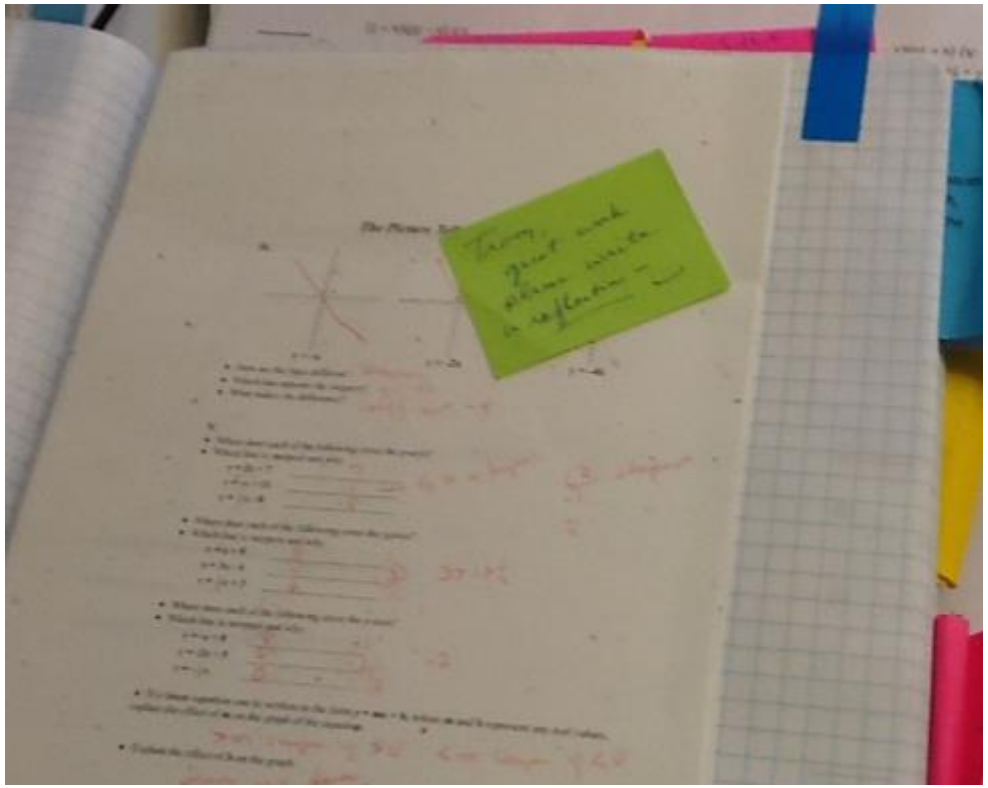

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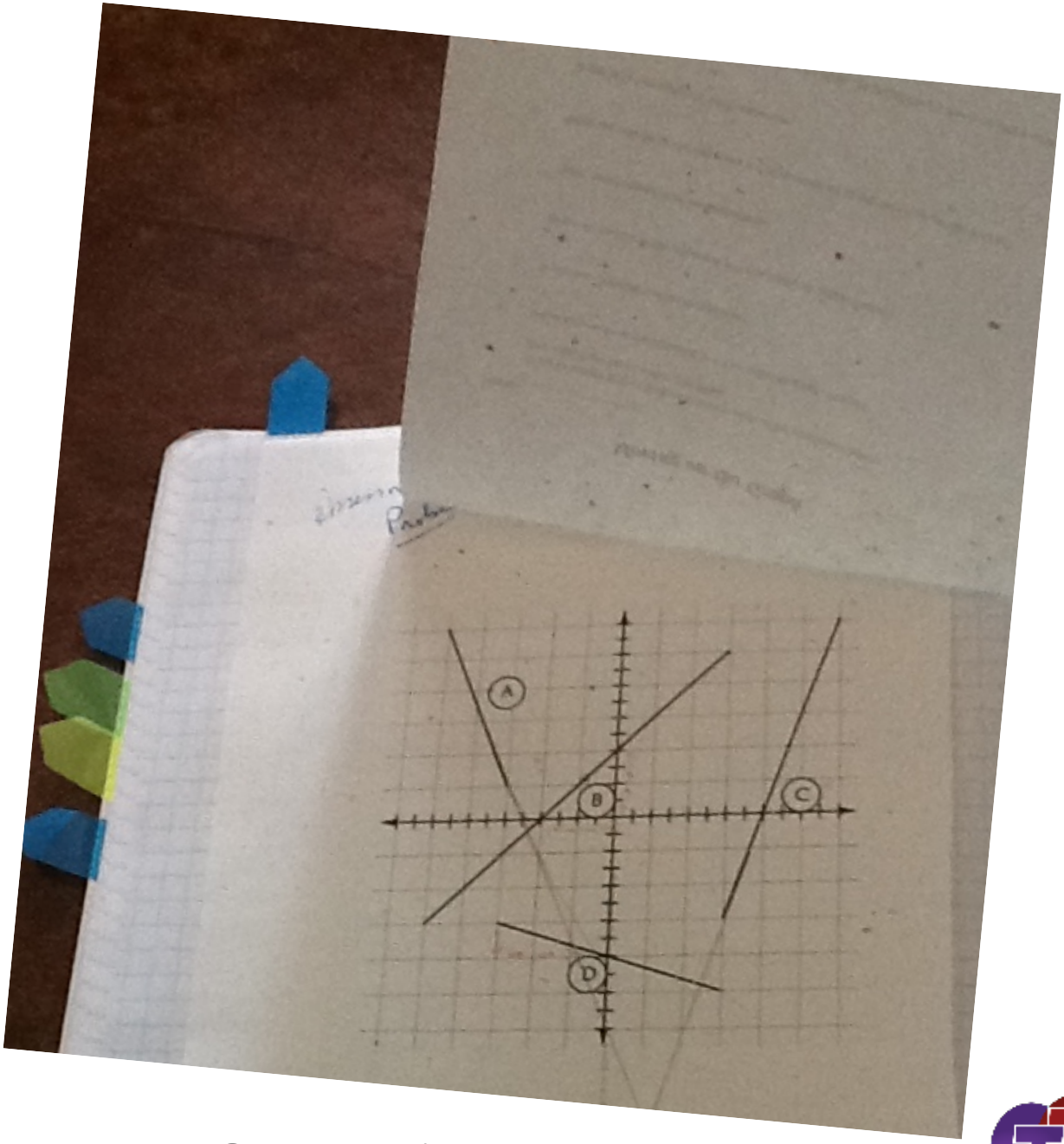
Assessment

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Glossary

Journal

Formulas/ Reference

Colored Page Test Notes

Calculator Notes

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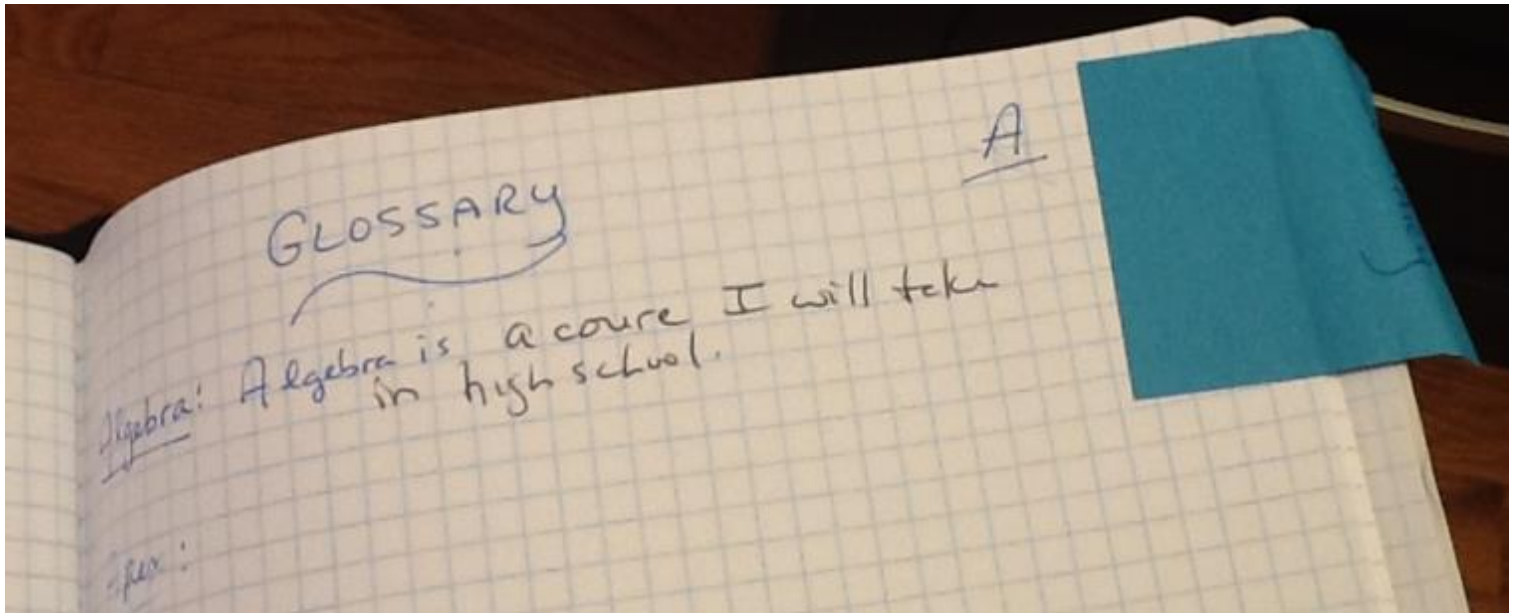
Appendices

# Glossary

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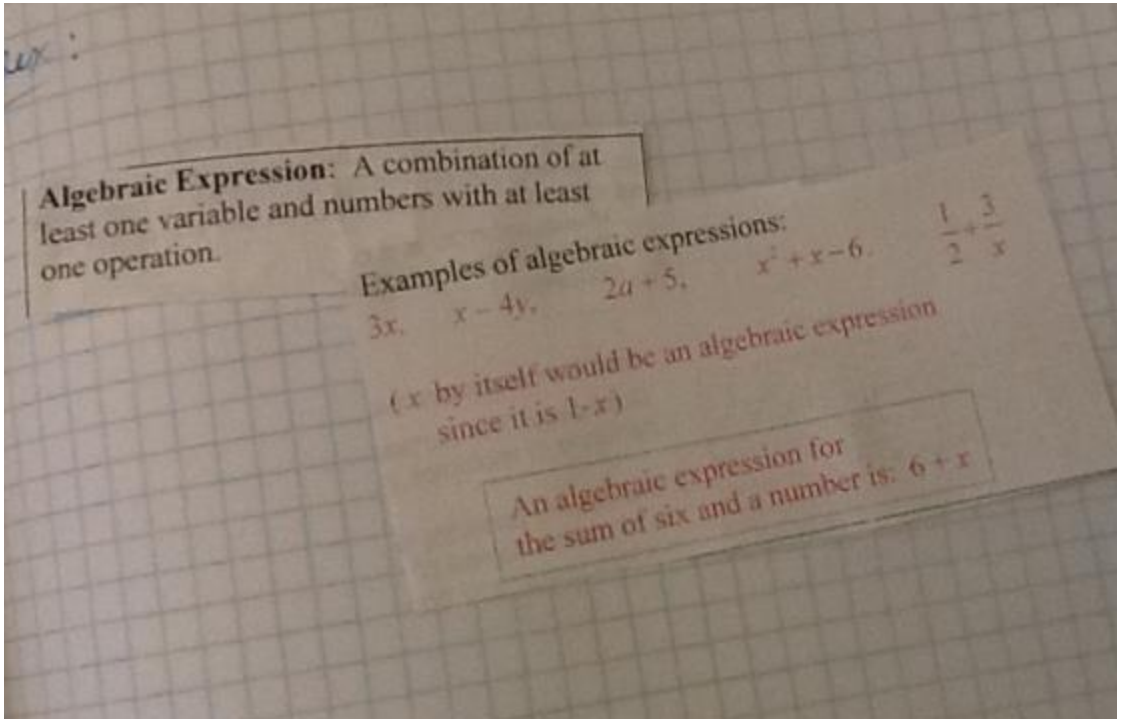
Appendices

# Glossary

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Appendices

# Journal

## Quotes

- Famous Mathematicians
- Cultural

## Special

- Math Autobiography
- Self-Evaluation
- Teacher Evaluation

## Math Specific

- Misconceptions
- HW Problem global challenges
- Short I-Search for set up
- Prompts “What if?”

Mac Tutor

Life’s Highway

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Appendices

Journal

"I apologize for the length of this letter. Had I but more time I would have made it shorter." Blaise Pascal

if?"

Mac Tutor

Life's Highway

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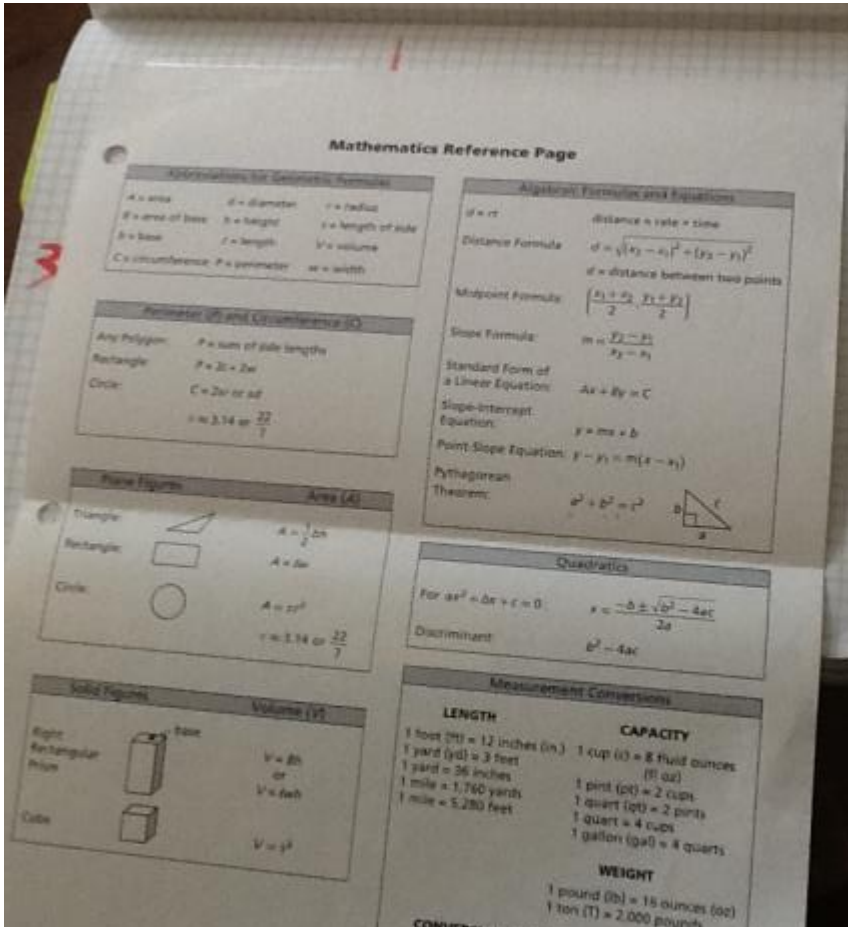
Appendices

# Formulas/ Reference

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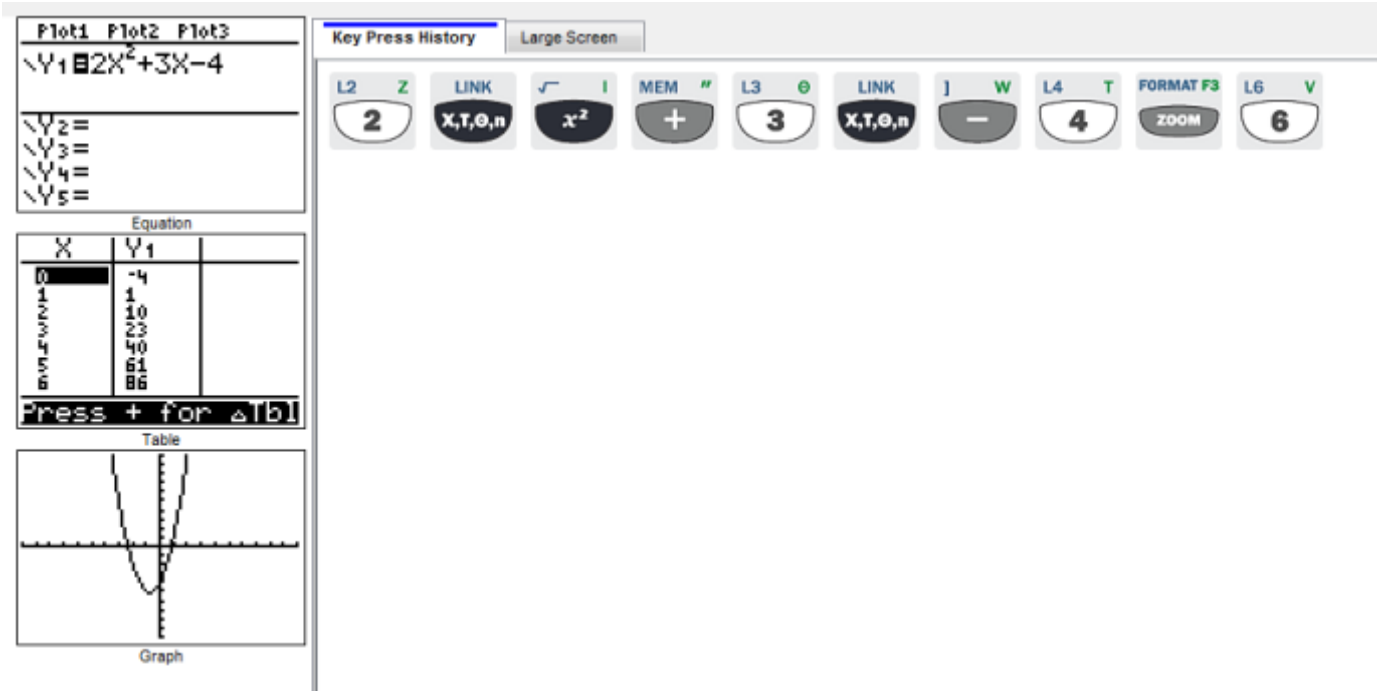
# Calculator Notes

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Appendices

Colored Page Test Notes

| Vocabulary | Discussion | Mathematics |
| --- | --- | --- |
| • I think you need to know to be mathematically culturally literate | • Fly on the wall…<br>• 2-D and 3-D graphing | • Procedural<br>• Check in for understanding |

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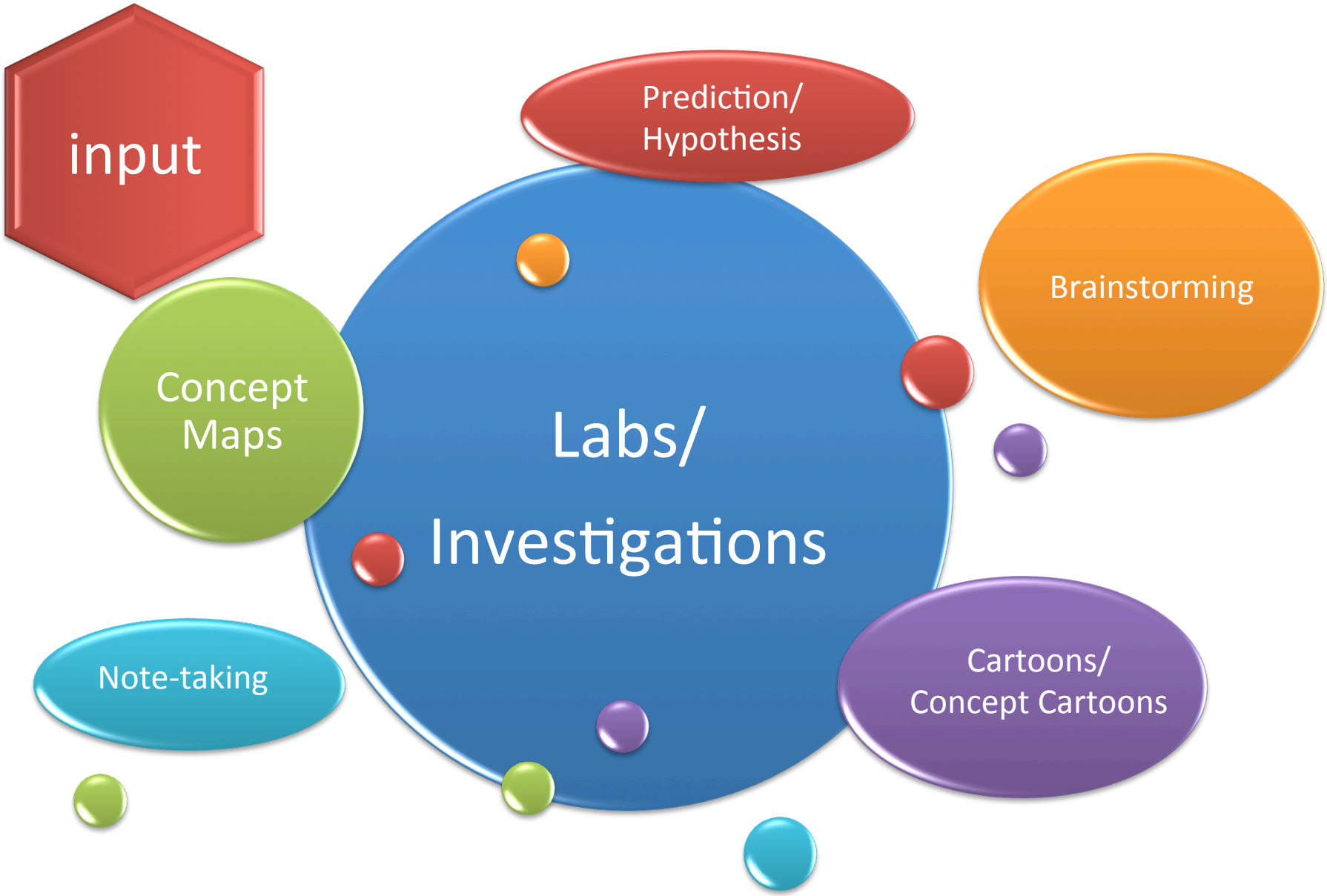


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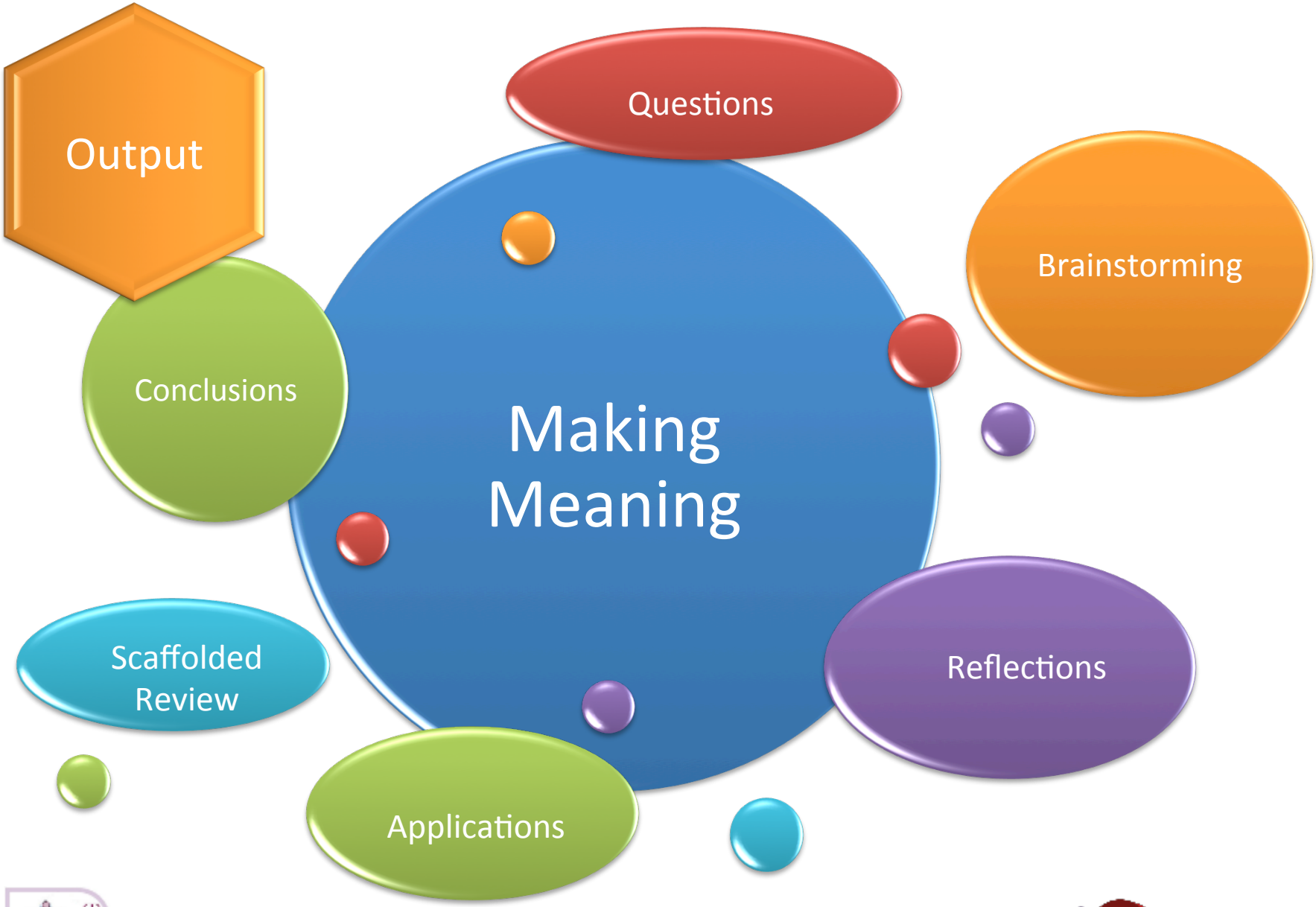


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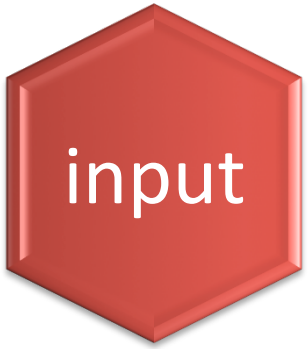


# A task…

- Label the top of your next even number page…

  "The Real Number System"

- Using the resources you have, create a model representation of the Real Number System, labeling and identifying all parts while attending to precision.

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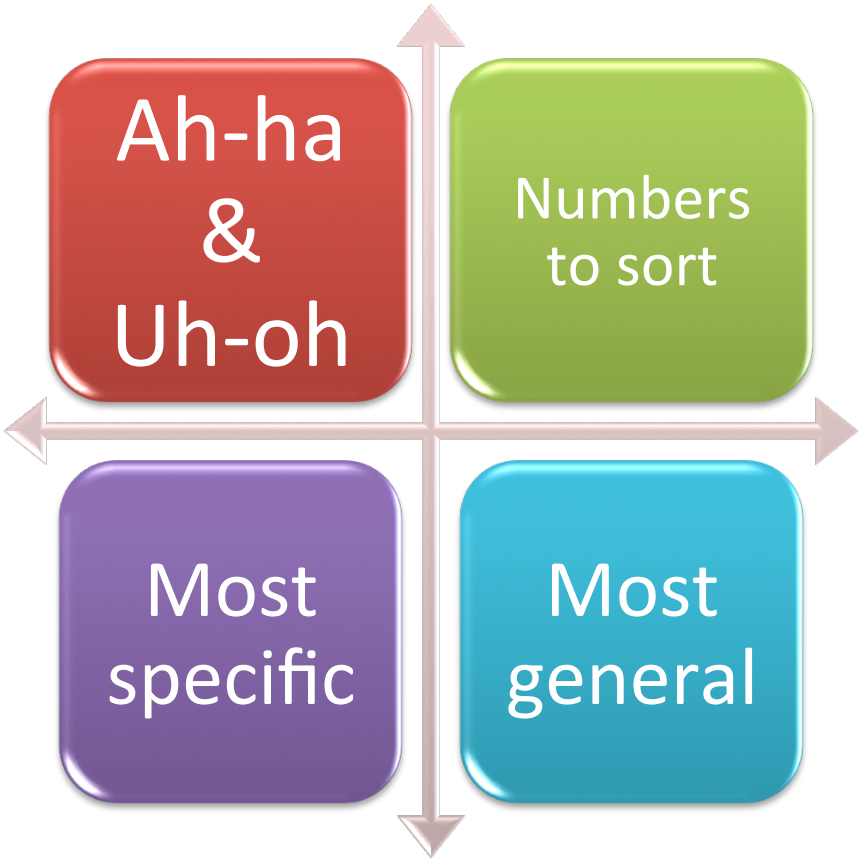
Output

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# Formative Assessment - Spiraling Review

Output

Identify each statement as being Always true, Sometimes true, or Never true. Justify your choice.

1. Whole numbers are rational.

2. Rational numbers are natural.

3. Integers are whole numbers.

4. A real number is an irrational number.

5. The difference in two whole numbers is a whole number.

6. The product of two integers is a natural number.

7. Fractions are rational numbers.

8. Make up a statement for a classmate and ask them what they think. Record your answer first, then record their answer.

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# Real Number System:

CLE 3102.2.2 Understand properties of and relationships between subsets and elements of the real number system.

√ 3102.2.7 Identify the subsets in the real number system and understand their relationships.

SPI 3102.2.3 Describe and/or order a given set of real numbers including both rational and irrational numbers.

**Use properties of rational and irrational numbers.**

CCSS.Math.Content.HSN-RN.B.3 Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.

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input

a. Complete the addition table.

| | 5 | 1/2 | 0 | $\sqrt{2}$ | $-\sqrt{2}$ | $\pi$ |
|---|---|---|---|---|---|---|
| **5** | 10 | 5.5 | | | | |
| **1/2** | | | | | | |
| **0** | | | | | | |
| $\sqrt{2}$ | | | | | | |
| $-\sqrt{2}$ | | | | | | |
| $\pi$ | | | | | | |

b. Complete the multiplication table.

| | 5 | 1/2 | 0 | $\sqrt{2}$ | $\frac{1}{\sqrt{2}}$ | $\pi$ |
|---|---|---|---|---|---|---|
| **5** | 25 | 2.5 | | | | |
| **1/2** | | | | | | |
| **0** | | | | | | |
| $\sqrt{2}$ | | | | | | |
| $\frac{1}{\sqrt{2}}$ | | | | | | |
| $\pi$ | | | | | | |

Illustrative Mathematics

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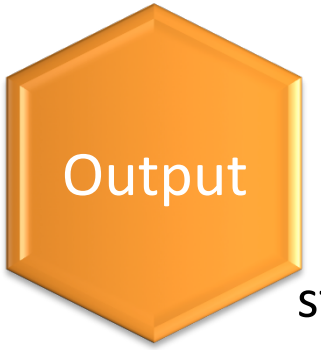


# Part C

Based on the above information, conjecture which of the statements is ALWAYS true, which is SOMETIMES true, and which is NEVER true?

i. The sum of a rational number and a rational number is rational.

ii. The sum of a rational number and an irrational number is irrational.

iii. The sum of an irrational number and an irrational number is irrational.

iv. The product of a rational number and a rational number is rational.

v. The product of a rational number and an irrational number is irrational.

vi. The product of an irrational number and an irrational number is irrational.

Illustrative Mathematics

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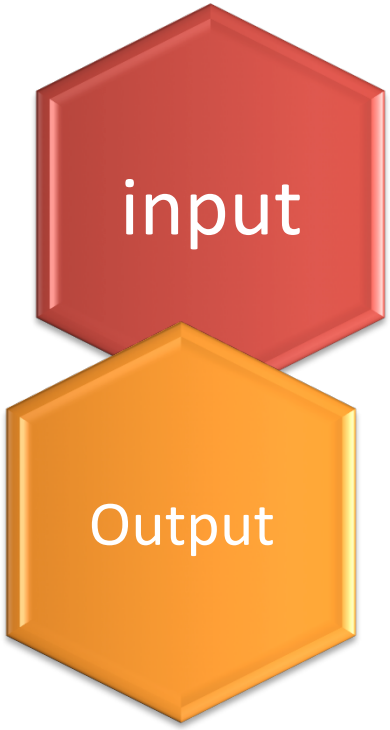


## Mathematics Assessment Program

## College and Career Readiness Mathematics

### Short Tasks: The Real Number System 1

1. Simplify $(\sqrt{2} - \sqrt{3})^2$
2. Simplify $(3\sqrt{5} - 2\sqrt{3})(3\sqrt{5} + 2\sqrt{3})$
3. Write $2.7 \times 10^4 + 120$ in scientific notation.

MARS Mathematics Assessment Resource Service

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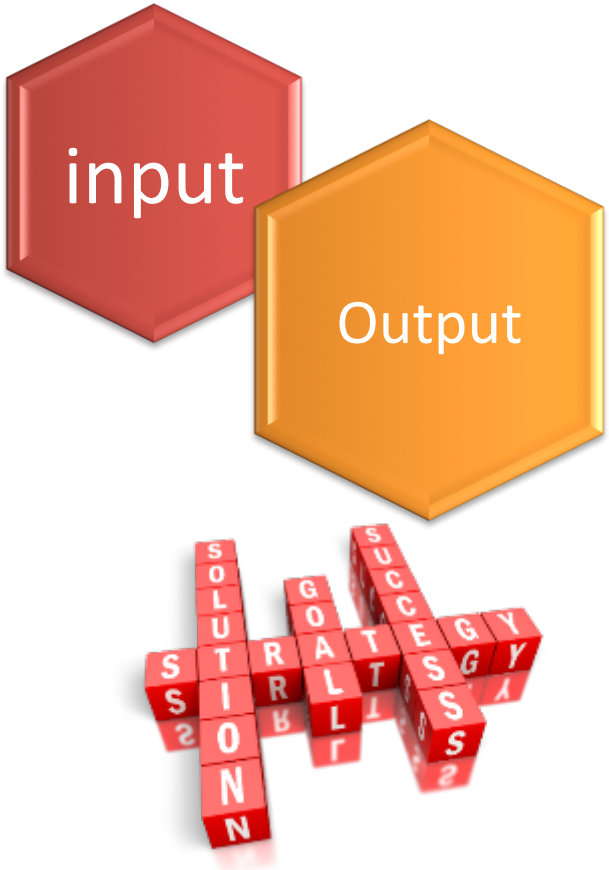

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Output

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input

The Complex Number System

- **Questions**
  - What properties are valid for all complex numbers?
  - How can you quickly evaluate $i$ raised to any power?
- **Journal/Writing Prompts**
  - Compare and contrast field properties of real numbers versus imaginary numbers.
  - Explain why the product of conjugates is always a real and rational number.
- **Investigations** (Identify the complex number described by each of the following descriptions, and verify your answer.)
  - The identity for addition on $C$
  - The multiplicative identity on $C$
  - The additive inverse of $a + bi$
  - The multiplicative inverse of $a + bi$
- **Representations** (Give the representation of each of the following statements in symbols, and state whether the statement is true or false. If false, give a counterexample. If true, verify (prove) the statement.)
  - The sum of two imaginary numbers is imaginary.
  - The product of two imaginary numbers is real.
  - Addition of complex numbers is commutative.
  - Multiplication of complex numbers is associative.
  - The distributive property of multiplication over addition holds for complex numbers.
  - The set of imaginary numbers is closed under addition.
  - The set of complex numbers is closed under multiplication.

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input

## Functions

On the grid are eight points from two different functions.

A certain linear function passes through exactly four of the points shown.

A certain quadratic function passes through the remaining four points.

For the **linear** function:

1. Write the coordinate pairs of its four points.

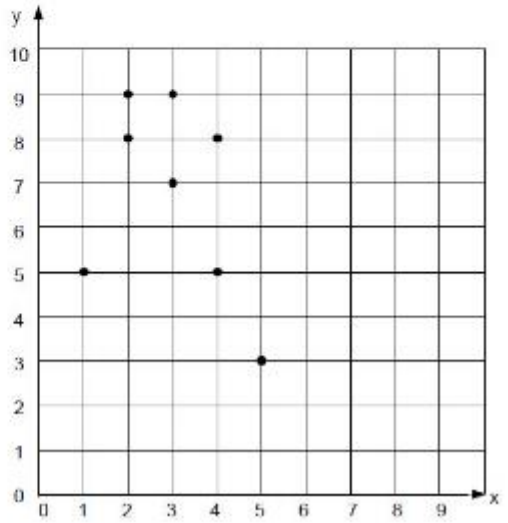
_____

_____

_____

_____

Draw the line on the grid.




MARS Mathematics Assessment Resource Service

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input

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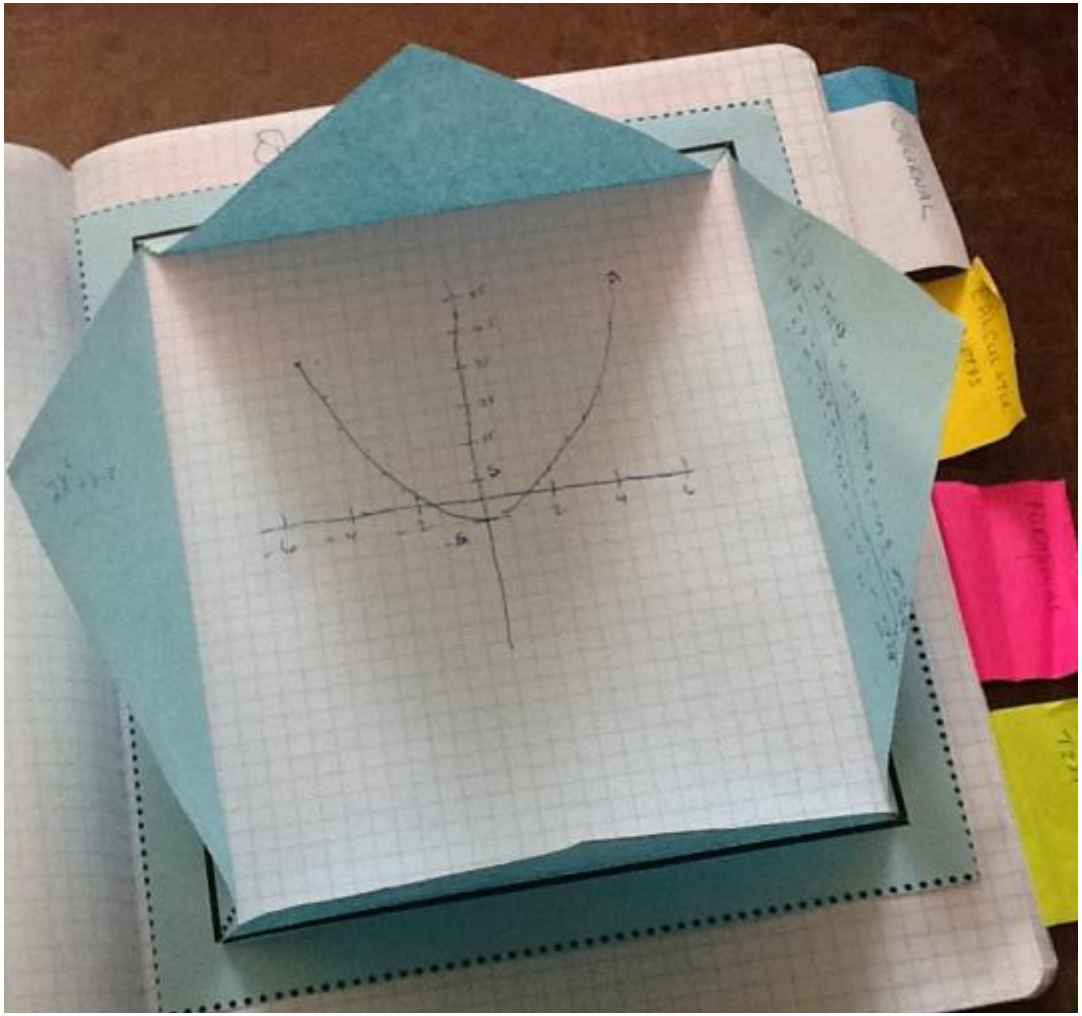
# More ideas



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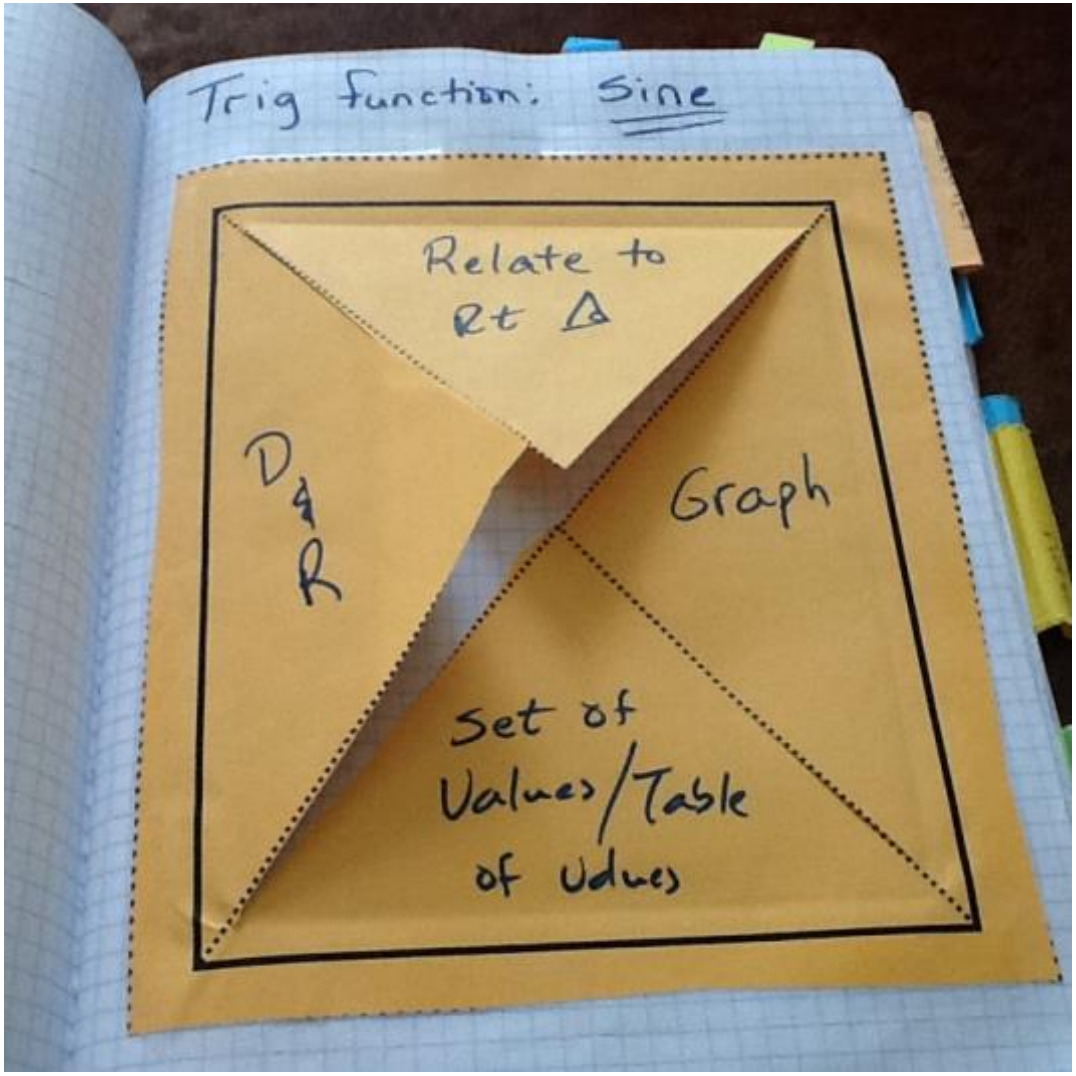
More ideas

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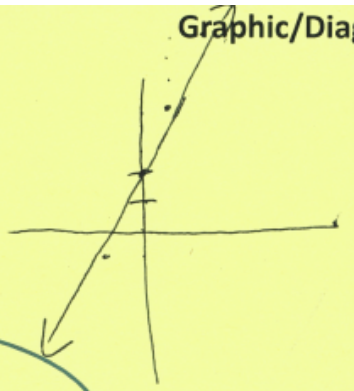
Output

**Words**

Two more than three times a number.

**Graphic/Diagrammatic**



$f(x) = 3x + 2$

**Set**

$\{(-2,-4),(-1,-1),(0,2)\}$

$\{x \mid x \in f(x) = 3x+2\}$

**Tabular**

| x | f(x) |
|---|---|
| -2 | -4 |
| -1 | -1 |
| 0 | 2 |
| 1 | 5 |
| 2 | 8 |

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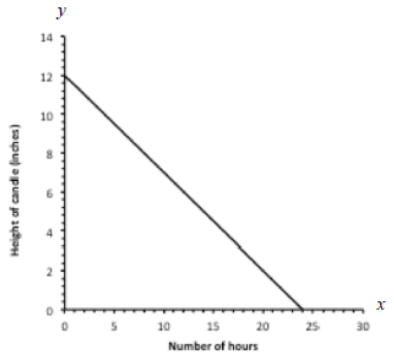
# Output

**Assessment task: *Four Situations***

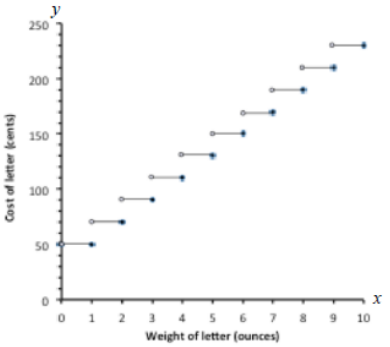
**A: Candle**

1. The candle sketch graph should be linear and decreasing as shown. Both time and height are continuous variables.
2. It corresponds to the function $y = 12 - 0.5x$
3. It will burn out completely after 24 hours.




**B: Letter**

1. The graph should be linear and increasing and passing through a point (0,A) where A>0.
2. Some students may realize that the $x$-axis is continuous and the $y$-axis will be discrete. Also the costs will jump as shown here.
3. The cost of a letter weighing x ounces is given by the function: $y = 30 + 20x$
4. A letter weighing 8 ounces will cost 190 cents.

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Journal

# Formative Assessment Anticipation Guide

Teacher: _____ FAL Name: _____

Grade: _____ Date: _____

**In anticipating the student work where will students demonstrate success?**

| What parts of the task will students demonstrate success? | In terms of knowing and doing mathematics what does this indicate? |
|---|---|
| | |

**In anticipating the student work where will students struggle (OMG's)?**

| What parts of the task will students be unsuccessful? | In terms of knowing and doing mathematics what does this indicate? What understandings or skills do the students need to learn? |
|---|---|
| | |

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Journal

# Analyzing Student Data

**FAL Title:** _____ **Teacher Name:** _____

**Date of Pre-Lesson Assessment:** _____ **Date of Post-Lesson Assessment:** _____

| *Analyzing Pre-Lesson Assessment Data* | | | | |
|---|---|---|---|---|
| Pre-Lesson Assessment Data | Demonstrates understanding **(3)** | Demonstrates some understanding **(2)** | Demonstrates little to no understanding **(1)** | No responses provided **(0)** |
| Number of Students | | | | |
| *What does the work reveal about the students' current level of understanding? What are the common issues? Summarize the difficulties by writing a series of questions.* | | | | |

| *Analyzing Post-Lesson Assessment Data* | | | | |
|---|---|---|---|---|
| Post-Lesson Assessment Data | Demonstrates understanding **(3)** | Demonstrates some understanding **(2)** | Demonstrates little to no understanding **(1)** | No responses provided **(0)** |
| Number of | | | | |

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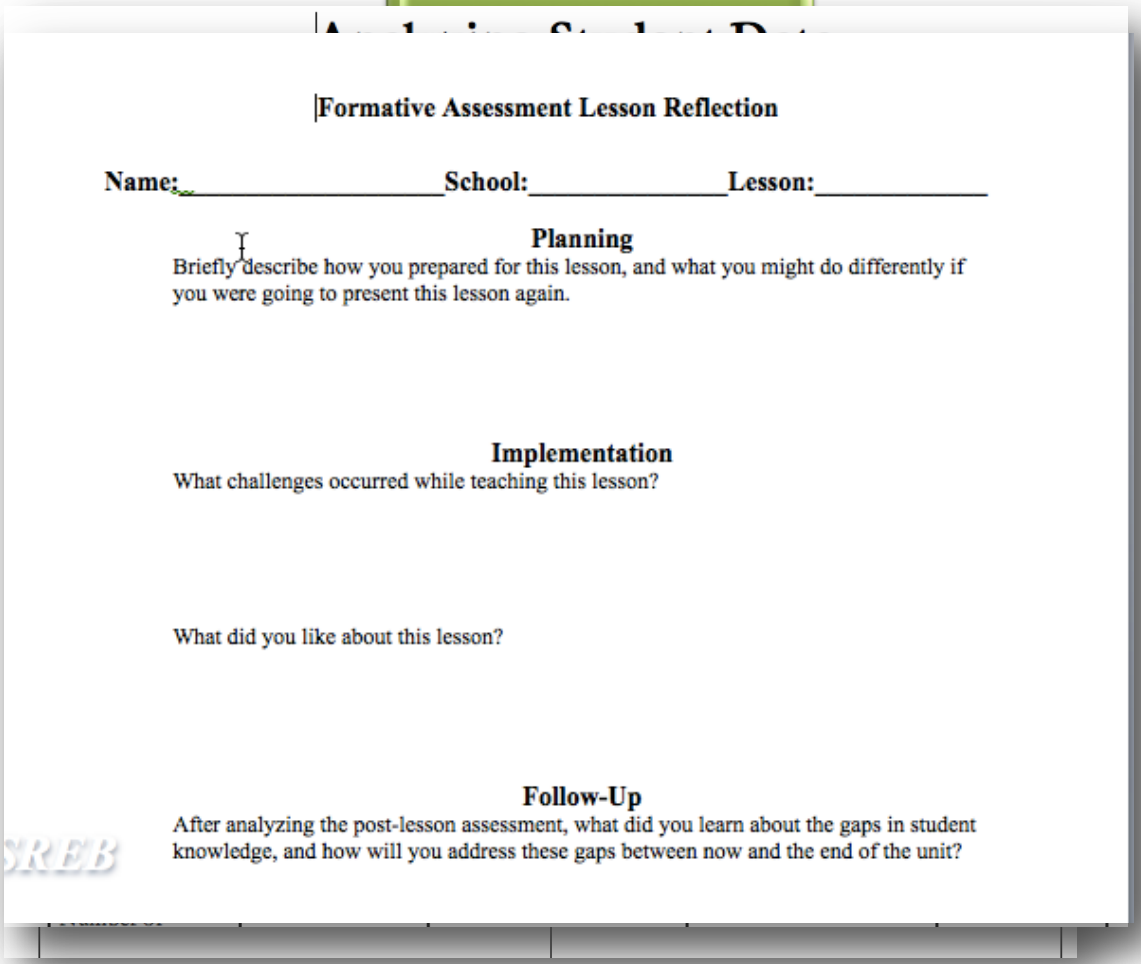
# Journal



**Formative Assessment Lesson Reflection**

**Name:_____School:_____Lesson:_____**

**Planning**

Briefly describe how you prepared for this lesson, and what you might do differently if you were going to present this lesson again.

**Implementation**

What challenges occurred while teaching this lesson?

What did you like about this lesson?

**Follow-Up**

After analyzing the post-lesson assessment, what did you learn about the gaps in student knowledge, and how will you address these gaps between now and the end of the unit?

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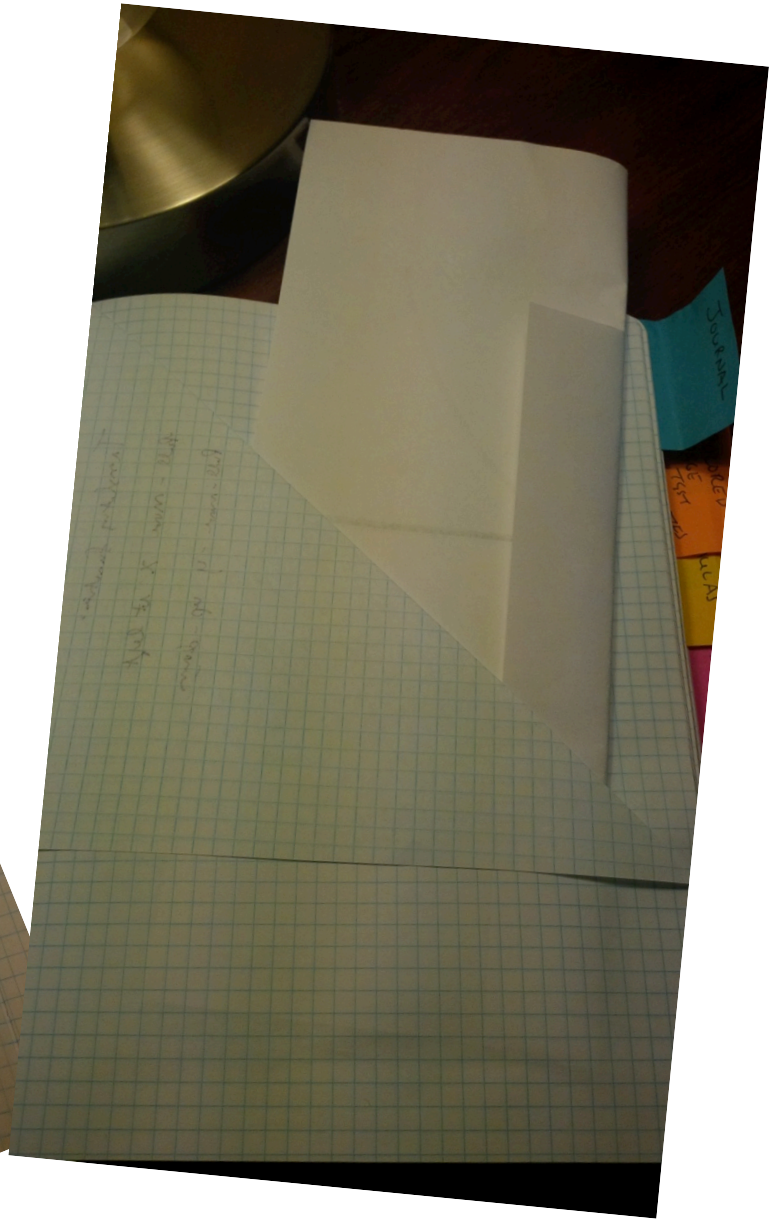
# More ideas



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Save the Date:

## October 5-6, 2015

Nashville, TN

## The MNB Secondary Workshop

An intensive 2-day workshop that will target vocabulary, writing and content.

For more information:

info@TLJConsultingGroup.com

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# Texas & Jones Networking Sessions

**Tuesday Sessions:**

**8:00-10:15** **A Mathematicians Notebook: Chronicling the Journey**

**10:30-11:30** **Effective Questioning to Build Deeper Understanding**

***All sessions are in Hilton 204-205**

**Wednesday Sessions:**

**9:30-10:45** **Mathematical Literacy in the Classroom: Using Trade Books**

****Session is in Hilton 202**

# Texas & Jones *HSTW* Sessions

**Thursday Sessions:**

| | |
|---|---|
| 8:00-9:00 | Effective Questioning to Build Deeper Understanding (Salon A Hilton) |
| 2:30-3:00 | Author Book Signing (Exhibit Hall- Routledge Booth) |
| 3:30-4:30 | Effective Questioning to Build Deeper Understanding (Salon A Hilton) |

**Friday Sessions:**

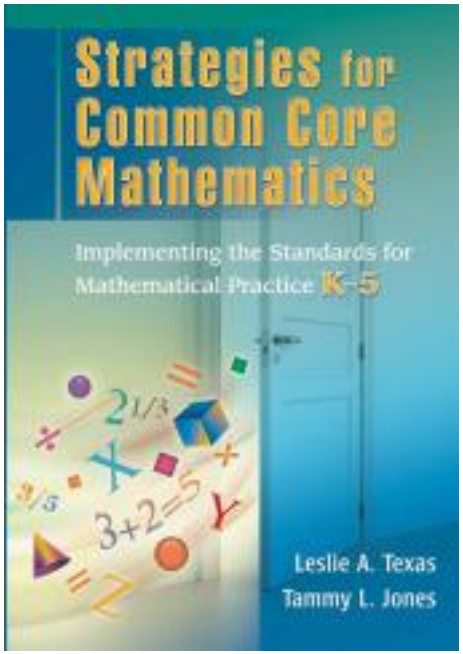
| | |
|---|---|
| 8:00-9:00 | Project-Based Learning: Meeting Rigorous Standards |
| 9:15-10:15 | Four Opportunities for Effective Questioning |
| 10:30-11:30 | Writing Across the Curriculum: The Strategic Journey Notebook |
| 2:15-4:30 | Authentic Opportunities for Reading and Writing in the Classroom |

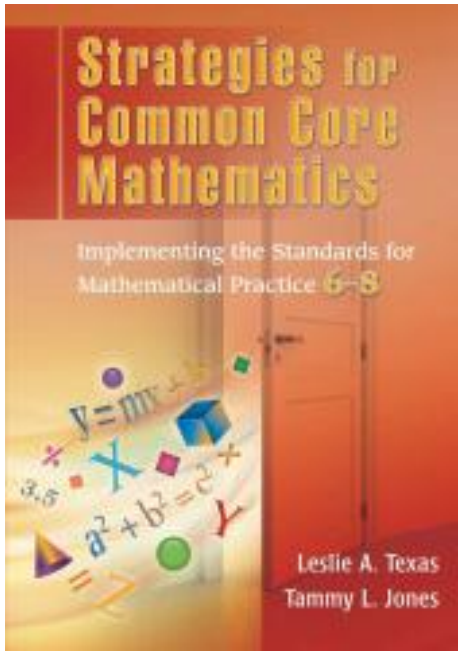
**Saturday Sessions:**

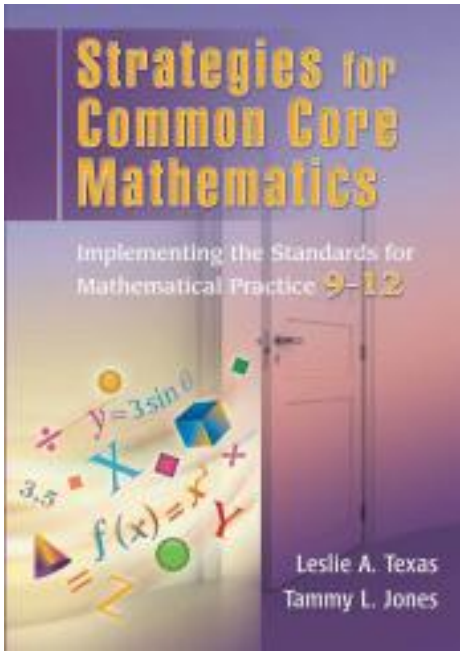
| | |
|---|---|
| 8:00-9:00 | Mathematical Literacy in the Classroom: Using Trade Books |
| 9:15-10:15 | Literacy & Mathematics in the CTE Classroom: Using Trade Books |

****All sessions on Friday and Saturday are in M103-105 (Marriott)**

# Book signing Thursday at 2:30 pm at the Routledge/Eye On Education Booth 408, 410

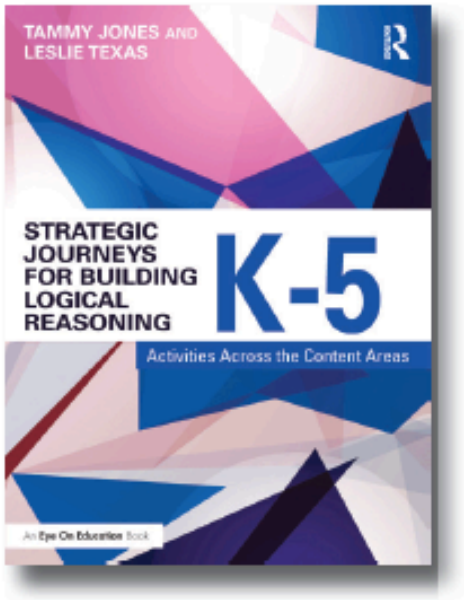


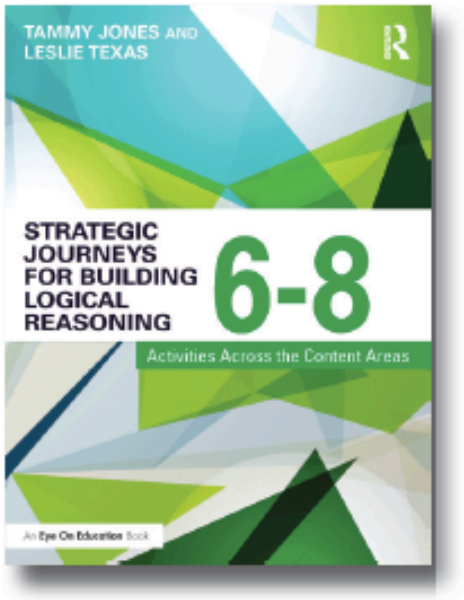



# Coming Spring 2016

## Strategic Journey's for Building Logic Reasoning



# A Mathematician's Notebook: Chronicling the Journey

Tammy L Jones
TammyJones@TLJConsultingGroup.com
Tammy@TexasAndJones.com
Leslie A. Texas
Leslie@TexasAndJones.com

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